Macroeconomic Fundamentals and Exchange Rate Dynamics: A No-Arbitrage Multi-Country Model

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Abstract

This paper investigates the relationship between exchange rate dynamics and macroeconomic fundamentals in a multi-country framework. Using a no-arbitrage macro-finance model, the exchange rate is a nonlinear function of macroeconomic fundamentals, which are assumed to be determined by global and country-idiosyncratic factors. The empirical study focuses on an open economy including four countries, i.e. Germany, the UK, Japan and the US (the home country). The model is able to characterize the joint dynamics of exchange rates, with 57%, 66% and 33% of the variations in the observed changes of USD/DEM (EUR), USD/GBP and USD/JPY being explained. All three dollar foreign exchange risk premia implied by this arbitrage-free multi-country model satisfy the Fama conditions (1984), they are highly correlated with each other and they are counter-cyclical with respect to the US economy. Moreover, global and country-idiosyncratic macroeconomic factors do exist and play very different roles in driving exchange rate dynamics and foreign risk premia.

Keywords: Multi-Country Model, Exchange Rate Dynamics, Macroeconomic Fundamentals, Global and Country-idiosyncratic Factors

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I. Introduction

The nominal floating exchange rate is of great interest both to academics and market practitioners. It has often been considered as the market price of one currency converted into another since the demise of the Bretton Woods system. According to present value models, its current price should reflect market's expectations concerning present and future macroeconomic conditions (Frenkel, 1981; Frenkel and Mussa, 1985; Obstfeld and Rogoff, 1996; Cochrane, 2005). However, a long-standing puzzle in international macroeconomics and finance is the disconnection between exchange rate dynamics and macroeconomic fundamentals (Obstfeld and Rogoff, 2001).

A large range of models have been proposed to link exchange rate dynamics to macroeconomic fundamentals, for instance, monetary models (Frenkel, 1976, 1979; Mussa, 1976; Bilson, 1978; Dornbusch, 1976), portfolio balance models (Dornbusch and Fischer, 1980; Campbell and Viceira, 2002), and new open economy macroeconomics models (Obstfeld and Rogoff, 2003). However, empirical studies using these models fail to provide evidence of a close connection between short-run exchange rate movements and macroeconomic fundamentals (Meese, 1990; Frankel and Rose, 1995; Sarno and Taylor, 2002; Engel and West, 2005). In general, these models imply that exchange rates are linear functions of macroeconomic fundamentals, which fail to capture the nonlinearity in exchange rate adjustments (Meese and Rose, 1991; Bleaney and Mizen, 1996; Obstfeld and Taylor, 1997). Moreover, they fail to capture the time-varying foreign exchange risk premium, implied by the well documented forward premium anomaly in foreign exchange markets (Fama, 1984; Hodrick, 1989; Backus, Gregory and Telmer, 1993; Bansal et al., 1995; Bekaert, 1996).

In this paper, I re-examine the exchange rate disconnect puzzle by proposing an arbitrage-free multi-country model that jointly prices exchange rates and bond yields using the information of macroeconomic fundamentals. Based on the equilibrium in the financial market, this arbitrage-free model links exchange rate dynamics to the macroeconomic fundamentals in a nonlinear way. Meanwhile, the foreign exchange risk premium is well structured by this arbitrage-free model. Under the principle of no-arbitrage, the exchange rate between any two countries is determined by the ratio of their stochastic discount factors, which are generally modeled by a factor representation in the asset pricing literature (Cochrane, 2005). The factors in determining the stochastic discount factor are the global and country-idiosyncratic macroeconomic factors, which are coming from the underlying macroeconomic fundamentals, i.e. output growth, inflation and short-term interest rates. Real output growth is taken as a key element of the stochastic discount factor (Ang and Piazzesi, 2003) for the reason that it directly dominates the aggregate consumption of an economy. Inflation comes into the stochastic discount factor because of its dynamic interactions with the real production (Piazzesi and Schneider, 2006). As a macroeconomic variable indicating the monetary policy, the short-term interest rate is included in the stochastic discount factor as well (Duffee, 2007).

I extend the close-economy term structure models in the macro-finance literature (Ang and Piazzesi, 2003; Diebold, Rudebusch and Aruoba, 2005; Ang, Dong and Piazzesi, 2007) to an open economy framework to price the foreign currency. In doing so, I can improve the identification of the time-varying market prices of risks by incorporating extra information from yield data. The time-varying market prices of risks are crucial in determining the foreign exchange risk premium and in amplifying roles of macroeconomic innovations on exchange rate movements. This is important since ignoring the foreign exchange risk premium or assuming constant foreign exchange risk premium may mislead to a conclusion that exchange rates are not linked to macroeconomic fundamentals.

A multi-country framework is used in this paper. It is important when studying exchange rate dynamics. First, with comparison to the two-country framework, it is much closer to the real world. It avoids the inconsistency issues concerning the parameters related to the numeraire country inherent in the two-country model when studying each single exchange rate separately. Furthermore, it is more helpful in exploiting the information based on positive correlations among dollar exchange rates (Hodrick and Vassalou, 2002).

In order to maintain the tractability of this multi-country model, the global and country-idiosyncratic factor setting is employed to represent the information of the underlying macroeconomic fundamentals. This setting has several advantages. First, it can highlight the fact of the cross-county comovement of the macroeconomic variables. Second, it is able to efficiently exploit the macroeconomic information and in the mean time to significantly reduce the number of the market prices of risk parameters. Hence, it maintains the feasibility in estimating this multi-country model. Third, I can distinguish the roles of the two different types of macroeconomic sources (i.e. global and country-idiosyncratic ones) in driving the exchange rate variations and the foreign exchange risk premium, which is novel in the literature.

Based on the above modeling setup, the exchange rate has a nonlinear relation with the underlying macroeconomic fundamental factors. In contrast to uncovered interest parity, this model indicates that the expected exchange rate changes are determined by both the interest rate differential and the foreign exchange risk premium and that the unexpected exchange rate changes are driven by the macroeconomic innovations, whose roles are amplified by the time-varying market prices of macroeconomic risks. According to the two different types of sources of the underlying macroeconomic fundamentals, global and country-idiosyncratic ones, the foreign exchange risk premium can be decomposed into two components, the global and country-idiosyncratic components.

Using monthly data of Germany, the UK, Japan and the US (the US being taken as the home country) ranging from January 1985 to May 2009, I find the empirical evidence of a close connection between macroeconomic fundamentals and the exchange rate dynamics. This no-arbitrage multi-country model is able to well characterize the joint dynamics of all the three major dollar exchange rates. The model-implied monthly exchange rate changes can explain 57%, 66% and 33% of the variations of the observed changes of USD/DEM (EUR), USD/GBP and USD/JPY, respectively. These findings are in stark contract to previous studies using monetary and new open economy macroeconomics models, with at most 10% of variation of the data being explained (Lubik and Schorfheide, 2005; Engel and West, 2005). Nevertheless, these findings are along the lines of recent studies of Dong (2006) and Li and Yin (2010). The former uses two macroeconomic variables and one latent and finds that 38% of variation of the dollar Mark exchange rate is explained. The later finds that 57% of variation of the dollar euro exchange rate is explained.

The model-implied foreign exchange risk premia are valid in explaining the forward premium anomaly as a consequence of the satisfactory of the Fama conditions (1984). The negative correlations of the short-term interest rate differentials and foreign exchange risk premia are consistent with the carry trade strategy in foreign exchange markets. Furthermore foreign exchange risk premia are counter-cyclical to the US economy, which is in line with De Santis and Fornari (2008) and Sarno, Schneider and Wagner (2011). This implies when the US economy is relatively better compared to foreign economies, the risk premia tend to depreciate foreign currencies and to appreciate the US dollar. The empirical study also reveals the high positive correlations among the risk premia of the three dollar foreign exchange rates, which is coherent with the fact that dollar exchange rates are positive correlated among each other. This finding is consistent with the results in Bams, Walkowiak, and Wolf (2004).

The macroeconomic innovations, or "news", are important in generating large amount of fluctuations of exchange rate dynamics. Moreover, their roles are amplified after taking into account of the nonlinearity. In this paper, the empirical evidence shows that the innovations account for around between 10% and 30% of the variations of the three dollar exchange rates. These results are in line with the findings using a different approach, the micro structure approach, with 30% of the explained variation in Evans and Lyons (2008) using the daily price of DEM/USD and 23% in Evan (2010) using the weekly USD/EUR. Furthermore, after magnifying the macroeconomic innovations by the corresponding time-varying market prices of risks, the proportion of the explained variations increases to around between 20% and 50% .

The global and country-idiosyncratic macroeconomic factors do exist and play very different roles in driving exchange rate dynamics and foreign exchange risk premia. The global factors drive foreign exchange risk premia almost exclusively and account for more than half of the variation of exchange rate dynamics, in both short and long runs. In the short run global interest rate is the dominant factor, while in the long run global output becomes dominant in driving both exchange rate dynamics and foreign exchange risk premia. Even though the country-idiosyncratic macroeconomic factors are less important compared to the global ones, they do play some role in determining the short-run exchange rate dynamics, especially interest rate factors of the US and Germany, and output growth factors of the UK and Japan.

This paper is relevant to previous studies on the joint dynamics of exchange rates and term structures of interest rates (Bansal, 1997; Backus, Foresi and Telmer, 2001; Dong, 2006; Marcello and Taboga, 2009; Ang and Chen, 2010; Han, Hammound and Ramezani, 2010; Li and Yin, 2010; Graveline and Joslin, 2011; Sarno, Schneider and Wagner, 2011). However, it is different in the following critical respects. First, I exclusively introduce macroeconomic fundamental information to modeling exchange rates in a multi-country framework. While Dong (2006) employs not only macroeconomic variables but also latent factors; Li and Yin (2010) use only macroeconomic variables but in a two-country framework; Sarno, Schneider and Wagner (2011) adopt latent factors. As a result, this study is able to link the exchange rate dynamics back to the macroeconomic fundamental. Second, I employ the global and country-idiosyncratic macroeconomic factors to investigate the joint dynamics of multiple exchange rates. Graveline and Joslin (2011) and Sarno, Schneider and Wagner (2011) apply global factors as well, however their global factors are latent. Third, this study sheds new light on the relationship between macroeconomic fundamentals and the exchange rate dynamics. Macroeconomic fundamentals enter into the exchange rate dynamics in a nonlinear form, through which the unexpected macroeconomic innovations play a crucial role in driving exchange rates and in generating large amount of variations of exchange rate changes. Fourth, this study illuminates the different roles played by global and country-idiosyncratic macroeconomic factors in driving exchange rate dynamics and foreign exchange risk premia. In the empirical evidence shows that global factors are the dominent ones in driving both exchange rate dynamics and foreign risk premia.

The rest of this paper is organized as follows. Section II describes the data. Section III introduces a multi-country no-arbitrage exchange rate model from a macro-finance perspective. Section IV proposes the econometric methodology, the likelihood-based estimation combined with the unscented Kalman filter. Section V presents the empirical results and discusses their economic implications. Section VI concludes.

II. Data and Preliminary Analysis

A. Data

Consider a multi-country world, with $(N+1)$ countries. The last country (the $(N+1)^{th}$ country) is the domestic country and the first N countries are the foreign countries. A $(3+1)$-country open economy case will be analyzed in the empirical study of this paper. The countries are Germany, the UK, Japan and the US, where the US is taken as the home country. The data is in the monthly time frequency and the sample period ranges from January 1985 to May 2009.

The macroeconomic fundamentals taken into account are output growth, inflation rates and short term interest rates. The nominal exchange rates are the end-of-the-period market rates. Both exchange rates and macroeconomic data are coming from the International Financial Statistics (IFS) database, provided by the International Monetary Fund (IMF).

Output growth rates and inflation rates are the one-year percentage changes of seasonal adjusted Industrial Production Indices (line *66*) and Consumer Price Indices (line *64*), respectively. Exchange rate data are the US dollar per national currency (line *ag*). The exchange rate for the German mark after 1999 is replaced by the exchange rate of the Euro, following Corte, Sarno and Tsiakas (2009).

— Figure 1 around here —

Moreover, yield data are included in order to better identify the parameters that determine the market prices of risks, since the market prices of risks are important in modeling exchange rate dynamics. The zero-coupon bond yield data for Germany, the UK, Japan and the US are taken from the International Zero Coupon Yield Curve Dataset used by Wright (2011). I take yields with different maturities: 3 months (the shortest one in this dataset), 24 months and 60 months, which stand for the short, medium and long term yields, respectively. These three yields are commonly used to obtain the empirical "level", "slope" and "curvature" components, which are sufficient to capture the term structure of interest rates. In addition, short-term interest rates are proxied by 3-month zero-coupon bond yields. In order to match the unit of monthly exchange rate movements, both the macroeconomic and yield data are divided by 12 so as to obtain monthly equal quantities.

B. Preliminary Analysis

It is important to clarify the cross-country relationships of macroeconomic fundamentals. The correlation matrix of these variables is presented in Table 8.

— Table 8 around here —

The diagonal sub-matrices of this correlation matrix point out relevant correlations among the three groups of macroeconomic variables (output growth, inflation and short-term interest rates). The first 4×4 triangular matrix on the diagonal of this correlation matrix shows that the output growths are all positively correlated across these four countries. Among them, the two highest correlations are between Germany and Japan (53%), and between the UK and the US (53%). The second 4×4 triangular matrix on the diagonal of this matrix shows that the inflation rates are positively correlated as well. The highest correlation is between the UK and the US (78%), followed by the UK and Japan (70%). The third 4×4 triangular matrix on the diagonal of the same matrix shows that short-term interest rates are also positively correlated, with the two highest correlations equal to 84% and 80% between the UK and Japan and between the UK and the US, respectively. The fact that macroeconomic variables are positively correlated across countries suggests that there may exist some global macroeconomic factors driving the cross-country comovement of output growth, inflation and short-term interest rates.

— Table 9 around here —

The above result is a starting point to explore deeper into the question of whether some common factors driving the cross-country comovement of macroeconomic fundamentals exist. Principle components analysis is conducted for each group of macroeconomic fundamentals (output growth, inflation and short-term interest rates). The results are reported in Table 9. In each group of macroeconomic fundamentals, the first principle component associated with the highest eigenvalue is able to explain 76%, 71% and 84% of the variations, respectively. This result implies that there should exist a global (common) factor in each group, determining the comovement of these macroeconomic variables across countries. These results provide clear evidence that global and country-idiosyncratic macroeconomic factors should be included in the setting of this model.

III. A Multi-Country No-Arbitrage Exchange Rate Dynamic Model

Consider a (*N+1*)-country world, with N foreign countries and *1* domestic country. Because of the important role of the US economy and its currency in the global economy activities after the Bretton Woods system collapse, the US is chosen as the home country and, correspondingly, the US dollar is the numeraire currency. Among these *N+1* countries' currencies, the sufficient amount of bilateral nominal exchange rate relationships is N, since the others can be deduced by the triangle relationship among these N bilateral exchange rates. Therefore, this paper focuses on the N bilateral exchange rates, which are the rates of the N foreign currencies against the US dollar.

The exchange rate dynamics are determined by the ratio of the stochastic discount factors between the home and the foreign country, following the no-arbitrage assumption and the law of one price. In this section, I first discuss the global and country-specific factor setting in a multi-country economy in subsection A. Next, I explain how to model stochastic discount factors which are determined by macroeconomic fundamentals in subsection B. After that, I proceed to model the exchange rate dynamics in subsection C. Finally, I present the recursive relationship which characterizes the bond pricing for each country under the affine term structure modeling framework in subsection D.

A. A Global and Country-Idiosyncratic Macro Factor Setting in a Multi-Country Economy

The choice of the state dynamics driving a multi-country economic system is a tradeoff. On one side, it is better to include as much macroeconomic information as possible. On the other, it is necessary to keep the amount of parameters as low as possible in order to be able to carry out the estimation. For this reason the macroeconomic global and country-idiosyncratic factors are introduced, since they are able to balance these two points.

Suppose there exist three global macroeconomic factors in a (*N+1*)-country economy, output growth g_t^G, inflation π_t^G, and the short-term interest rate r_t^G, which drive the comovement of macroeconomic fundamentals across countries. Let G_t be a vector of global factors, where $G_t = \left(g_t^G,\ \pi_t^G,\ r_t^G\right)^T$.

For each economy i ($i = 1, 2, ..., N+1$), I assume that its underlying macroeconomic fundamental vector $X_{i,t} = \left(\tilde{g}_{i,t},\ \tilde{\pi}_{i,t},\ \tilde{r}_{i,t}\right)^T$ loads on the global factor vector $G_t = \left(g_t^G,\ \pi_t^G,\ r_t^G\right)^T$, as well as on its country-idiosyncratic factor vector $F_{i,t} = \left(f_{i,t}^g,\ f_{i,t}^\pi,\ f_{i,t}^r\right)^T$. Note that the tilde is used to distinguish between the unobserved underlying fundamentals and the observed data with the difference being measurement errors. Hence, for country i, its underlying macroeconomic fundamentals $\tilde{g}_{i,t}$, $\tilde{\pi}_{i,t}$ and $\tilde{r}_{i,t}$ are,

$$
\begin{aligned}
\tilde{g}_{i,t} &= \alpha_i^g + \beta_i^g g_t^G + f_{i,t}^g, \\
\tilde{\pi}_{i,t} &= \alpha_i^\pi + \beta_i^\pi \pi_t^G + f_{i,t}^\pi, \\
\tilde{r}_{i,t} &= \alpha_i^r + \beta_i^r r_t^G + f_{i,t}^r,
\end{aligned} \tag{1}
$$

where $\{\alpha_i^g, \alpha_i^\pi, \alpha_i^r\}_{i=1,\ldots,N+1}$ are constant terms, and $\{\beta_i^g, \beta_i^\pi, \beta_i^r\}_{i=1,\ldots,N+1}$ are country i's loadings on global factors (g_t^G, π_t^G, r_t^G); $\{f_{i,t}^g, f_{i,t}^\pi, f_{i,t}^r\}_{i=1,\ldots,N+1}$ are country-idiosyncratic factors of country i. Rewriting the above equations into matrix form,

$$X_{i,t} = \alpha_i + \beta_i G_t + F_{i,t}, \tag{2}$$

where $\{\alpha_i\}_{i=1,\ldots,N+1}$ are constant 3×1 vectors, and $\{\beta_i\}_{i=1,\ldots,N+1}$ are diagonal matrices of the loadings on G_t.

Notice that G_t and $F_{i,t}$ are two different types of state vectors and together they determine the underlying macroeconomic fundamentals. The global factor vector G_t is assumed to follow a Gaussian vector autoregressive process,

$$G_t = \Phi^G G_{t-1} + \Sigma^G \upsilon_t^G, \tag{3}$$

where Φ^G is a constant 3×3 matrix; υ_t^G is an i.i.d. Gaussian white noise, with zero mean and an identity variance-covariance matrix; Σ^G is a diagonal matrix. In order to identify the global factors, two sets of assumptions are needed. First, since the magnitude of global factors and their loadings cannot be separately identified, I assume that the innovations related to global factors have a standard deviation of 0.001, which means $\Sigma^G = 0.001 \times I_3$. Second, I assume that the US loadings on the global factors are positive, in order to identify the signs of the global factors and their loadings.

The country-idiosyncratic factor vector $F_{i,t}$ is assumed to have a Gaussian vector autoregressive process,

$$F_{i,t} = \Phi^{F_i} F_{i,t-1} + \Sigma^{F_i} \upsilon^F_{i,t}, \tag{4}$$

where Φ^{F_i} is a constant 3×3 diagonal matrix; $\upsilon^F_{i,t}$ is country-idiosyncratic shock vector, with zero mean and an identity variance-covariance matrix. I assume that shocks in this equation are independent, hence the variance-covariance matrix of $\Sigma^{F_i}(\Sigma^{F_i})^T$ is diagonal. A similar setting with global and country-idiosyncratic factors associated to "level" and "slope" is used by Diebold, Li, and Yue (2008) to investigate the global yield curve in a multi-country economy.

B. Relating Macroeconomic Fundamentals to Stochastic Discount Factors

In this multi-country world, I assume that the no-arbitrage condition holds. Then there exists at least one almost surely positive process M_t with $M_0 = 1$ denominated in each currency, such that the discounted gain process associated with any admissible trading strategy is a martingale (Harrison and Kreps (1979)). M_t is called the stochastic discount factor (*SDF*). I denote the country i's SDF as $M_{i,t}$, for $i = 1, 2, ..., N + 1$.

Since there is no widely accepted general equilibrium model for asset pricing, I follow those studies that choose a partial equilibrium approach to model financial markets and use flexible factor models with the no-arbitrage condition (Cochrane, 2004). In this paper, I use a factor representation for the SDF's, which allows us to model both exchange rates and term structures of interest rates. Under the complete market assumption, there exists one unique stochastic discount factor $M_{i,t}$, associated with each country i's currency, for $i = 1, 2, ..., N+1$. Given that the dynamics of country i's economy are jointly determined by the global factor as well as by its country-idiosyncratic factor, I assume that the SDF for country i has the following exponential form,

$$\begin{aligned} M_{i,t+1} &= \exp(m_{i,t+1}) \\ &= \exp\Big(-\tilde{r}_{i,t} - \frac{1}{2}(\lambda^G_{i,t})^T \lambda^G_{i,t} - \frac{1}{2}(\lambda^F_{i,t})^T \lambda^F_{i,t} - (\lambda^G_{i,t})^T \upsilon^G_{t+1} - (\lambda^F_{i,t})^T \upsilon^F_{i,t+1}\Big), \end{aligned} \tag{5}$$

where $\tilde{r}_{i,t}$ is the short-term interest rate of country i; $\lambda^G_{i,t}$ and $\lambda^F_{i,t}$ are the time-varying market prices of global and country-idiosyncratic risks assigned by investors to those assets denominated in country i's currency; υ_{t+1}^{G} and $\upsilon_{i,t+1}^{F}$ are the global and country-idiosyncratic "uncertainties" related to the country i's economy at time t, defined by equations (3) and (4).

This specification for the SDF process is similar to the one commonly used in the macro finance term structure literature (Ang and Piazzesi (2003), Duffee (2002) and Duffee (2007)). The only difference from the standard ones is that in this paper there are two types of market prices of riskss and innovations associated with two types of state vectors, global and country-idiosyncratic ones. The stochastic discount factor is also named the intertemporal marginal rate of substitution in a Lucas-type exchange economy (Lucas (1982)), which is derived by solving the representative agent's optimization problem.

Note that the market prices of global and country-idiosyncratic risks related to country i's currency are $\lambda_{i,t}^{G}$ and $\lambda_{i,t}^{F}$, respectively. The country i's state vectors G_t and $F_{i,t}$ summarize the uncertainties in country i's economy. I assume that the market prices of global and country-idiosyncratic risk related to each country i's currency are affine functions of their corresponding state vectors, G_t and $F_{i,t}$, for each country $i = 1, ..., N+1$, (Dai and Singleton (2002); Duffee (2002)),

$$\lambda_{i,t}^{G} = \lambda_{i,0}^{G} + \lambda_{i,1}^{G} G_t, \tag{6}$$

$$\lambda_{i,t}^{F} = \lambda_{i,0}^{F} + \lambda_{i,1}^{F} F_{i,t}, \tag{7}$$

where $\lambda_{i,0}^{G}$ and $\lambda_{i,0}^{F}$ are constant 3×1 vectors, and $\lambda_{i,1}^{G}$ and $\lambda_{i,1}^{F}$ are constant 3×3 matrices. It is crucial to make some reasonable restrictions on the coefficient matrices, $\lambda_{i,1}^{G}$ and $\lambda_{i,1}^{F}$, in order to estimate the model. Here I simplify $\lambda_{i,1}^{G}$ and $\lambda_{i,1}^{F}$ to be diagonal matrices. By doing so, I are able to reduce the amount of parameters in this multi-country model without loss of generality and efficiency when modeling market prices of risks using macroeconomic information.

C. Exchange Rate Dynamics

Let $\mathcal{S}_{j,t}$ $(j = 1, ..., N)$ be the exchange rate between the foreign country j and the US, which is defined as the price of the US dollar per one unit of the foreign country j's currency. The no-arbitrage assumption and the law of one price imply that the ratio of the stochastic discount factors between the home and the foreign country determines the dynamics of their exchange rate (Bachus, Foresi, and Telmer (2001); Bekaert (1996); Brandt and Santa-Clara (2002); Brandt, Cochrane, and Santa-Clara (2006)). Thus I have,

$$\frac{\mathcal{S}_{j,t+1}}{\mathcal{S}_{j,t}} = \frac{M_{j,t+1}}{M_{N+1,t+1}}. \tag{8}$$

The above relation formally defines the link between the stochastic discount factors of two economies and the exchange rate movements between them. In complete markets, the stochastic discount factors in both economies are unique, therefore they uniquely determine the dynamics of their exchange rate.

Taking natural logarithms of both sides of equation (8) and using the specification of the SDF (equation (5)), I obtain the following equation for exchange rate changes,

$$\begin{aligned}\Delta s_{j,t+1} \quad = \quad & \Big(\tilde{r}_{N+1,t} - \tilde{r}_{j,t}\Big) + \frac{1}{2}\Big((\lambda^G_{N+1,t})^T \lambda^G_{N+1,t} - (\lambda^G_{j,t})^T \lambda^G_{j,t} + (\lambda^F_{N+1,t})^T \lambda^F_{N+1,t} - (\lambda^F_{j,t})^T \lambda^F_{j,t}\Big) \\ & + \Big((\lambda^G_{N+1,t})^T \upsilon^G_{t+1} - (\lambda^G_{j,t})^T \upsilon^G_{t+1} + (\lambda^F_{N+1,t})^T \upsilon^F_{N+1,t+1} - (\lambda^F_{j,t})^T \upsilon^F_{j,t+1}\Big),\end{aligned} \tag{9}$$

which shows that the global factor G and the two country-idiosyncratic factors, F_j for foreign country j and F_{N+1} for the home country, determine the exchange rate changes $\Delta s_{j,t+1}$, via market prices of risks in a nonlinear form. This is in contrast to the traditional models that often get a linear relation between the exchange rate dynamics and macroeconomic fundamentals or to other models that only use latent factors which do not have any economically meaningful interpretations.

The exchange rate changes can be divided into two components, the expected and the unexpected component. The expected change of the exchange rate is,

$$\begin{aligned}\Delta s^{exp.}_{j,t+1} \quad \equiv \quad & E_t\Big(\Delta s_{j,t+1}\Big) \qquad (10) \\ = \quad & \Big(\tilde{r}_{N+1,t} - \tilde{r}_{j,t}\Big) + \frac{1}{2}\Big((\lambda^G_{N+1,t})^T \lambda^G_{N+1,t} - (\lambda^G_{j,t})^T \lambda^G_{j,t}\Big) + \frac{1}{2}\Big((\lambda^F_{N+1,t})^T \lambda^F_{N+1,t} - (\lambda^F_{j,t})^T \lambda^F_{j,t}\Big),\end{aligned}$$

which captures the predictable variation of returns in foreign exchange markets. I can see that market prices of riskss are important in modeling the expected component of exchange rate changes. The uncovered interest rate parity does not hold for this model, since the expected exchange rate changes are determined by both the interest rate differentials between the two countries $(\tilde{r}_{N+1,t} - \tilde{r}_{j,t})$ and by a time varying foreign exchange risk premium term, $rp_{j,t+1}$,

$$rp_{j,t+1} \equiv \frac{1}{2}\Big((\lambda^G_{N+1,t})^T \lambda^G_{N+1,t} - (\lambda^G_{j,t})^T \lambda^G_{j,t}\Big) + \frac{1}{2}\Big((\lambda^F_{N+1,t})^T \lambda^F_{N+1,t} - (\lambda^F_{j,t})^T \lambda^F_{j,t}\Big), \tag{11}$$

The above equation shows that the foreign exchange risk premium is determined by two components, the one driven by global factors and the other driven by country-idiosyncratic factors.

The unexpected change of the exchange rate is,

$$\begin{aligned} \Delta s^{unexp.}_{j,t+1} &\equiv \Delta s_{j,t+1} - E_t\Big(\Delta s_{j,t+1}\Big) \qquad (12) \\ &= \Big((\lambda^G_{N+1,t})^T \upsilon^G_{t+1} - (\lambda^G_{j,t})^T \upsilon^G_{t+1}\Big) + \Big((\lambda^F_{N+1,t})^T \upsilon^F_{N+1,t+1} - (\lambda^F_{j,t})^T \upsilon^F_{j,t+1}\Big), \end{aligned}$$

It implies that the unexpected change of the exchange rate is composed by the products of state vector shocks times their corresponding market prices of risks. Similarly to the foreign exchange risk premium in equation (11), the unexpected change of the exchange rate also has two components, the global component and the country-idiosyncratic component. It can be noticed that the market prices of risks are time-varying, which are driven by the dynamics of the global factor and the country-idiosyncratic factor. This implies the exchange rate changes are heteroskedastic.

In summary, the equation for exchange rate dynamics (9) can also be written in the following way,

$$\begin{aligned} \Delta s_{j,t+1} &= \Big(\tilde{r}_{N+1,t} - \tilde{r}_{j,t}\Big) + rp_{j,t+1} + \Delta s^{unexp.}_{j,t+1}, \qquad (13) \\ &= \Delta s^{exp.}_{j,t+1} + \Delta s^{unexp.}_{j,t+1}, \qquad (14) \end{aligned}$$

D. Bond Pricing

In the last part of this model, I introduce the recursive relationship which characterizes the bond pricing for each country i $(i = 1, ..., N+1)$ under the affine term structure modeling framework.

For each country i, having specified the stochastic discount factor $M_{i,t}$ (equation (5)) and its state dynamics (equation (3) and (4)), I can price its zero-coupon bonds. Introducing bond information in this model is important to identify market prices of risks.

Each country's short rate $\tilde{r}_{i,t}$ is a function of the global factor as well as its country-idiosyncratic factor, as equation (1) shows. I can write the short rate equation as an affine function of the global factor G_t and its country-idiosyncratic factor $F_{i,t}$,

$$\tilde{r}_{i,t} = \delta_{i,0} + (\delta_{i,1}^G)^T G_t + (\delta_{i,1}^F)^T F_{i,t}, \tag{15}$$

with $\delta_{i,0} = \alpha_i^r$, $\delta_{i,1}^G = \left(0, 0, \beta_i^r\right)^T$, and $\delta_{i,1}^F = \left(0, 0, 1\right)^T$.

In each country i, the no-arbitrage condition guarantees that a zero-coupon bond with n-period maturity at time t can be priced according to the following Euler equation,

$$\tilde{P}_{i,t}^{(n)} = E_t\left[M_{i,t+1}\tilde{P}_{i,t+1}^{(n-1)}\right] \tag{16}$$

with the initial condition $\tilde{P}_{i,t}^{(0)} = 1$. As before, tilde indicates the true value.

By combining equations (3) and (4), which describe the dynamics of the factor vectors, together with equation (15) which defines the short rate and equation (5) which specifies the SDF, I can show that country i's bond price is an exponential linear function of the global factor G_t and of the country-idiosyncratic factor $F_{i,t}$,

$$\tilde{P}_{i,t}^{(n)} = \exp\Big(A_{i,n} + (B_{i,n})^T G_t + (C_{i,n})^T F_{i,t}\Big), \tag{17}$$

where $A_{i,n}, B_{i,n}$ and $C_{i,n}$ solve the following difference equations,

$$\begin{aligned} A_{i,n+1} &= A_{i,n} - (B_{i,n})^T \Sigma^G \lambda_{i,0}^G - (C_{i,n})^T \Sigma^{F_i} \lambda_{i,0}^F + \frac{1}{2}(B_{i,n})^T \Sigma^G (\Sigma^G)^T B_{i,n} + \frac{1}{2}(C_{i,n})^T \Sigma^{F_i} (\Sigma^{F_i})^T C_{i,n} - \delta_{i,0}, \\ B_{i,n+1} &= \left(\Phi^G - \Sigma^G \lambda_{i,1}^G\right)^T B_{i,n} - \delta_{i,1}^G, \\ C_{i,n+1} &= \left(\Phi^{F_i} - \Sigma^{F_i} \lambda_{i,1}^F\right)^T C_{i,n} - \delta_{i,1}^F, \end{aligned} \tag{18}$$

with $A_{i,1} = -\delta_{i,0}$, $B_{i,1} = -\delta_{i,1}^G$, and $C_{i,1} = -\delta_{i,1}^F$ being the initial conditions. Accordingly, the yield is also an affine function of the state

$$\tilde{y}_{i,t}^{(n)} \equiv -\frac{\log P_{i,t}^{(n)}}{n} = a_{i,n} + (b_{i,n})^T G_t + (c_{i,n})^T F_{i,t}, \tag{19}$$

where $a_{i,n} = -A_{i,n}/n$, $b_{i,n} = -B_{i,n}/n$, and $c_{i,n} = -C_{i,n}/n$.

From the difference equations (18), I can see that the constant coefficients of market price of risk $\lambda_{i,0}^G$ and $\lambda_{i,0}^F$ only affect the constant yield coefficient $a_{i,n}$; on the contrary, the parameters $\lambda_{i,1}^G$ and $\lambda_{i,1}^F$ affect the loadings on the global and the country-idiosyncratic factors, $b_{i,n}$ and $c_{i,n}$, respectively. This implies that the parameters $\lambda_{i,0}^G$ and $\lambda_{i,0}^F$ affect average term spreads and average expected bond returns, whereas the parameters $\lambda_{i,1}^G$ and $\lambda_{i,1}^F$ determine time variation in term spreads and expected bond returns.

IV. Econometric Methodology

I have assumed that the macroeconomic factors $X_{i,t}$, yields $y_{i,t}$ and exchange rate changes $\Delta s_{j,t}$ are unobservable and that the econometrician observes the corresponding ones, $X_{i,t}^{obs.}$, $y_{i,t}^{obs.}$ and $\Delta s_{j,t}^{obs.}$, with measurement errors, $\eta_{i,t}^X$, $\eta_{i,t}^y$ and $\eta_{j,t}^{\Delta s}$. I can first transform the model into a state-space representation and then use a Bayesian filtering approach to estimate it.

A. State-Space Model Representation

At each period t, I can observe the exchange rate changes, the macroeconomic variables and the zero-coupon bond data. I assume that each of these variables is collected with normal i.i.d measurement errors. Thus, I have the following measurement equations

$$\begin{aligned}
\Delta s_{j,t}^{obs.} &= \Big(\tilde{r}_{N+1,t-1} - \tilde{r}_{j,t-1}\Big) + \frac{1}{2}\Big((\lambda_{N+1,t-1}^G)^T \lambda_{N+1,t-1}^G - (\lambda_{j,t-1}^G)^T \lambda_{j,t-1}^G \\
&\quad + (\lambda_{N+1,t-1}^F)^T \lambda_{N+1,t-1}^F - (\lambda_{j,t-1}^F)^T \lambda_{j,t-1}^F\Big) + (\lambda_{N+1,t-1}^G - \lambda_{j,t-1}^G)^T (\Sigma^G)^{-1} (G_t - \Phi^G G_{t-1}) \\
&\quad + \Big((\lambda_{N+1,t-1}^F)^T (\Sigma^{F_{N+1}})^{-1} (F_{N+1,t} - \Phi^{F_{N+1}} F_{t-1}) - (\lambda_{j,t-1}^F)^T (\Sigma^{F_j})^{-1} (F_{j,t} - \Phi^{F_j} F_{t-1})\Big) + \eta_{j,t}^{\Delta s}, \\
&\quad \textit{for } j = 1, \ \dots\ , N; \qquad (20) \\
X_{i,t}^{obs.} &= \alpha_i + \beta_i G_t + F_{i,t} + \eta_{i,t}^X, \ \textit{for } i = 1, ..., N+1;, \qquad (21) \\
y_{i,t}^{obs.} &= a_i + (b_i)^T G_t + (c_i)^T F_{i,t} + \eta_{i,t}^y, \quad \textit{for } i = 1, \ \dots\ , N+1. \qquad (22)
\end{aligned}$$

where in equation (20), I use $\upsilon_t^G = (\Sigma^G)^{-1}(G_t - \Phi^G G_{t-1})$ and $\upsilon_{i,t} = (\Sigma^{F_i})^{-1}(F_{i,t} - \Phi^{F_i} F_{t-1})$ (for $i = 1, ..., N+1$), from equations (3) and (4); the market prices of risks are linear functions of the state vectors, $\lambda_{i,t-1}^G = \lambda_0^G + \lambda_1^G G_t$ and $\lambda_{i,t-1}^F = \lambda_0^F + \lambda_1^F F_{i,t}$. Measurement errors ($\eta_t$) are with distinct variances for different variables/series and are assumed to be mutually independent.

The state vectors in this multi-country system, which are the global factor G_t and the country-idiosyncratic factor $F_{i,t}$ (for $i = 1, 2, 3, 4$), follow a first-order VAR and their dynamics are described by equations (3) and (4). From the measurement equations I can notice that observations depend on both current and lagged values of the global and the country-idiosyncratic factors. Hence all of them should be taken as states and the state equations are the following,

$$\begin{pmatrix} G_t \\ G_{t-1} \end{pmatrix} = \begin{pmatrix} \Phi^G & 0_{3\times3} \\ I_3 & 0_{3\times3} \end{pmatrix} \begin{pmatrix} G_{t-1} \\ G_{t-2} \end{pmatrix} + \begin{pmatrix} I_3 \\ 0_{3\times3} \end{pmatrix} \Sigma^G \upsilon_t^G, \tag{23}$$

$$\begin{pmatrix} F_{i,t} \\ F_{i,t-1} \end{pmatrix} = \begin{pmatrix} \Phi^{F_i} & 0_{3\times3} \\ I_3 & 0_{3\times3} \end{pmatrix} \begin{pmatrix} F_{i,t-1} \\ F_{i,t-2} \end{pmatrix} + \begin{pmatrix} I_3 \\ 0_{3\times3} \end{pmatrix} \Sigma^{F_i} \upsilon_{i,t}^F, \; for \; i = 1, ..., N+1 \tag{24}$$

Therefore, the set of parameters of this multi-Country model which I have to estimate is,

$$\Theta = \Big(\{\alpha_i, \; \beta_i; \; \Phi^{F_i}, \; \Sigma^{F_i}; \; \lambda_{i,0}^G \, \lambda_{i,1}^G, \;\; \lambda_{i,0}^F \, \lambda_{i,1}^F; \; \Sigma^{\eta^{X_i}}, \; \Sigma^{\eta^{y_i}}\}_{i=1,2,3,4}; \; \{\sigma^{\eta^{\Delta s_j}}\}_{j=1,2,3}; \; \Phi^G, \; \Sigma^G \Big), \tag{25}$$

B. Quasi-Maximum Likelihood Estimation and Unscented Kalman Filter

Given that this state-space model representation (equations (20) to (24)) has Gaussian noises, I can implement the model estimation using Bayesian filtering approaches. The exchange rate dynamic equations are highly non-linear functions of the states, which makes the standard Kalman filter inapplicable. Instead, I can use the nonlinear Kalman filters. The most commonly used nonlinear Kalman filter is the extended Kalman filter, which linearizes the nonlinear system around the current state estimate using a Taylor approximation. However, for the highly nonlinear system, the extended Kalman filter is computationally demanding and performs very poorly. An alternative is the unscented Kalman filter (UKF), recently developed in the field of engineering (Julier and Ulman (1997, 2004)). The idea behind this approach is that in order to estimate the state information after a nonlinear transformation, it is better to approximate the probability distribution directly instead of linearizing the nonlinear functions. The unscented Kalman filter overcomes pitfalls inherent to the extended Kalman filter to a large extent and improves estimation accuracy and robustness without increasing computational cost.

In order to implement the unscented Kalman filter, I first concatenate the state variables $x_{t-1} = [G_{t-1}, F_{1,t-1}, ..., F_{4,t-1}, G_{t-2}, F_{1,t-2}, ..., F_{4,t-2},]'$, the observation noises η_{t-1} and the state noises $\varepsilon_{t-1} = [\upsilon^G_{t-1}, \upsilon^F_{1,t-1}, ..., \upsilon^F_{4,t-1}]'$ at time $t-1$,

$$x^e_{t-1} = \begin{bmatrix} x'_{t-1} & \eta'_{t-1} & \varepsilon'_{t-1} \end{bmatrix}', \tag{26}$$

whose dimension is $L = L_x + L_\eta + L_\varepsilon$ and whose mean and covariance are

$$\hat{x}^e_{t-1} = \begin{bmatrix} E[x_{t-1}] & 0 & 0 \end{bmatrix}', \qquad P^e_{t-1} = \begin{bmatrix} P^x_{t-1} & 0 & 0 \\ 0 & \Sigma^2_\eta & 0 \\ 0 & 0 & I_{15} \end{bmatrix}.$$

I then form a set of $2L+1$ sigma points

$$\chi^e_{t-1} = \begin{bmatrix} \hat{x}^e_{t-1} & \hat{x}^e_{t-1} + \sqrt{(L+\lambda)P^e_{t-1}} & \hat{x}^e_{t-1} - \sqrt{(L+\lambda)P^e_{t-1}} \end{bmatrix} \tag{27}$$

and the corresponding weights

$$w^{(m)}_0 = \frac{\lambda}{L+\lambda}, \quad w^{(c)}_0 = \frac{\lambda}{L+\lambda} + (1 - \alpha^2 + \beta), \tag{28}$$

$$w^{(m)}_i = w^{(c)}_i = \frac{1}{2(L+\lambda)}, \quad i = 1, 2, \ldots, 2L, \tag{29}$$

where superscripts (m) and (c) indicate that the weights are for construction of the posterior mean and of the covariance, respectively; $\lambda = \alpha^2(L+\bar{\kappa}) - L$ is a scaling parameter; the constant α determines the spread of sigma points around $\bar{x}$ and is usually set to be a small positive value; $\bar{\kappa}$ is a second scaling parameter with value set to 0 or $3-L$; β is a covariance correction parameter and is used to incorporate prior knowledge of the distribution of x.

With these sigma points, I implement the UKF as follows: for the time update

$$\begin{aligned}
\chi^x_{t|t-1} &= F(\chi^x_{t-1}, \chi^\varepsilon_{t-1}), \quad \hat{x}^-_t = \sum_{i=0}^{2L} w_i^{(m)} \chi^x_{i,t|t-1}, \\
P^-_{x_t} &= \sum_{i=0}^{2L} w_i^{(c)} (\chi^x_{i,t|t-1} - \hat{x}^-_t)(\chi^x_{i,t|t-1} - \hat{x}^-_t)',
\end{aligned}$$

and for the measurement update

$$\begin{aligned}
\mathcal{Y}_{t|t-1} &= H(\chi^x_{t|t-1}, \chi^\eta_{t|t-1}), \quad \hat{Y}^-_t = \sum_{i=0}^{2L} w_i^{(m)} \mathcal{Y}_{i,t|t-1}, \\
P^-_{Y_t} &= \sum_{i=0}^{2L} w_i^{(c)} (\mathcal{Y}_{i,t|t-1} - \hat{Y}^-_t)(\mathcal{Y}_{i,t|t-1} - \hat{Y}^-_t)', \\
P_{x_t Y_t} &= \sum_{i=0}^{2L} w_i^{(c)} (\chi^x_{i,t|t-1} - \hat{x}^-_t)(\mathcal{Y}_{i,t|t-1} - \hat{Y}^-_t)', \\
\hat{x}_t &= \hat{x}^-_t + P_{x_t Y_t}(P^-_{Y_t})^{-1}(Y_t - \hat{Y}^-_t), \\
P_{x_t} &= P^-_{x_t} - (P_{x_t Y_t}(P^-_{Y_t})^{-1}) P^-_{Y_t} (P_{x_t Y_t}(P^-_{Y_t})^{-1})',
\end{aligned}$$

where Y_t is the observation vector containing all the observed variables, $\hat{Y}^-_t$ its predicted values, $P^-_{Y_t}$ its conditional variance-covariance matrix, $\hat{x}_t$ the filtered state vector, and P_{x_t} its variance-covariance matrix.

Assuming that the predictive errors are normally distributed, I can construct the log likelihood function at time t as follows

$$\mathcal{L}_t(\Theta) = -\frac{1}{2} \ln |P^-_{Y_t}| - \frac{1}{2}(Y_t - \hat{Y}^-_t)'(P^-_{Y_t})^{-1}(Y_t - \hat{Y}^-_t), \tag{30}$$

where Θ is a parameter vector of the model. Parameter estimates can be obtained by maximizing the joint log likelihood

$$\hat{\Theta} = \arg\max_{\Theta \in \Xi} \sum_{t=1}^{T} \mathcal{L}_t(\Theta), \tag{31}$$

where Ξ is a compact parameter space, and T is the length of total observations of the data. Since the log likelihood function is misspecified for the non-Gaussian model, a robust estimate of the variance-covariance matrix of parameter estimates can be obtained using the approach proposed by White (1982)

$$\hat{\Sigma}_{\Theta} = \frac{1}{T}\left[AB^{-1}A\right]^{-1}, \tag{32}$$

where

$$A = -\frac{1}{T}\sum_{t=1}^{T}\frac{\partial^2\mathcal{L}_t(\hat{\Theta})}{\partial\Theta\partial\Theta'}, \quad B = \frac{1}{T}\sum_{t=1}^{T}\frac{\partial\mathcal{L}_t(\hat{\Theta})}{\partial\Theta}\frac{\partial\mathcal{L}_t(\hat{\Theta})}{\partial\Theta'}. \tag{33}$$

With these parameter estimates $\hat{\Theta}$, the latent global and country-idiosyncratic factors, $\hat{G}_t$ and $\hat{F}_{i,t}$ $(i = 1, 2, 3, 4)$, can be extracted using the unscented Kalman filter.

The number of parameters in this model is large. Maximization of the likelihood (30) may involve a large number of likelihood evaluations. Therefore, I adopt a sophisticated quasi-Newton approach with the inverse Hessian matrix of the likelihood function updated by the BFGS algorithm. The initial values are carefully selected in the following way. I first run the Nelder-Mead optimization algorithm for 100 feasible sets of starting values and stop them after 100 iterations. Then the best 10 parameter estimate sets (in terms of the likelihood) are selected among these 100 runnings as the initial values for the quasi-Newton algorithm. The parameter estimates are those comiing from the largest likelihood among these 10 runnings of the quasi-Newton method.

V. Empirical Results and Discussions

A. Model Performance on Exchange Rate Dynamics

Exchange rate dynamics are the main focus of this paper. The summary statistics from the observed and the model-implied data exchange rates dynamics are reported in Table 4. The model is able to capture the statistic moments of the observed exchange rate movements. However model-implied exchange rate changes are less volatile than the observed ones.

— Table 4 around here —

The above results are also confirmed by Figure 2, where both the model implied exchange rate dynamics and the observed ones are plotted. Generally speaking, the model implied exchange rate dynamics are able to reproduce the dynamics of the observed ones very well along the sample period. The first panel in Table 5 shows that the model-implied exchange rate dynamics are able to capture 57%, 66% and 33% of the variations of the observed exchange rate dynamics for the USD/DEM (EUR), the USD/GBP and the USD/JPY, respectively. Comparing with linear models of exchange rate dynamics that use macroeconomic fundamental information, this no-arbitrage multi-country model represents a big improvement. This finding is quite consistent with other studies based on alternative approaches. For instance, Evans and Lyons (2002) adopt a micro-market structure approach and obtain the R^2 statistics equal to 64% and 45% for daily German mark/dollar and Japanese yen/dollar log changes between May 1 to August 31, 1996.

— Figure 2 around here —

B. Macroeconomic Shocks and Exchange Rate Dynamics

Previous studies have found that exchange rate movements are largely disconnected with macroeconomic fundamentals. In monetary models and/or new open economy macroeconomic models, the exchange rate is a linear function of contemporaneous macroeconomic variables. Since the residuals are usually serially correlated in these models, the estimation is implemented using first-order differences of relevant variables,

$$\Delta s_t = \beta_0 + \beta_1^{(h)} \Delta r_t^{(h)} + \beta_1^{(f)} \Delta r_t^{(f)} + \beta_2^{(h)} \Delta g_t^{(h)} + \beta_2^{(f)} \Delta g_t^{(f)} + \beta_3^{(h)} \Delta \pi_t + \beta_3^{(f)} \Delta \pi_t^{(f)} + u_t, \quad (34)$$

where u_t is a noise term. In these models, coefficients are typically constrained by $\beta_k^{(h)} = -\beta_k^{(f)}$, for $k = 1, 2, 3$. When estimating this linear model for the three types of exchange rate changes used in this paper, I find R^2 equal to 3.3%, 5.7% and 4.5% for the unconstrained regressions and R^2 equal to 1.4%, 1.2% and 1.5% for the constrained regressions. Even though macroeconomic fundamentals in this model are able to account for 57%, 66% and 33% of the variation of exchange rate movements for the three dollar exchange rate dynamics respectively, the linear model in equation (34) cannot capture this link between macroeconomic fundamentals and exchange rates.

What exact roles do macroeconomic fundamentals play in this model? Recall the exchange rate dynamic equation (13),

$$\Delta s_{j,t+1} \quad = \quad \left(\tilde{r}_{N+1,t} - \tilde{r}_{j,t}\right) + rp_{j,t+1} + \Delta s^{unexp.}_{j,t+1},$$

where I decompose the exchange rate dynamics into three components, the short-term interest differential, the foreign exchange risk premium and the unexpected exchange rate changes.

— Figure 3 around here —

Figure 3 presents these three components of the exchange rate changes as well as their sum, the model-implied exchange rate changes. For each exchange rate change, the first component, $(\tilde{r}_{N+1,t} - \tilde{r}_{j,t})$, which is the only concern in the UIP model, is very smooth for all three dollar exchange rates. The second one, foreign exchange risk premium $rp_{j,t+1}$, becomes volatile in comparison to the first term, but it still has much smaller variation than the model-implied exchange rate changes. This implies that the third component, $\Delta s^{unexp.}_{j,t+1}$, must be more volatile and should play a more important role in explaining exchange rate movements. Figure 3 shows that $\Delta s^{unexp.}_{j,t+1}$ is very volatile and reproduces fluctuations of exchange rate changes.

From equation (12), the unexpected exchange rate change $\Delta s^{unexp.}_{j,t+1}$ has two parts: one is driven by global innovations $(\lambda^G_{N+1,t})^T \upsilon^G_{t+1} - (\lambda^G_{j,t})^T \upsilon^G_{t+1}$; the other is driven by country-idiosyncratic innovations $(\lambda^F_{N+1,t})^T \upsilon^F_{N+1,t+1} - (\lambda^F_{j,t})^T \upsilon^F_{j,t+1}$, both of which are macro-dependent. The regression of the data on the unexpected exchange rate changes and a constant results in an R^2 of 48% (57% or 21%), which equal to 84% (86% or 64%) of the total explained variance for the USD/DEM (the USD/GBP or the USD/JPY) by this model. However, when I regress the data on the model-implied macroeconomic innovations (or macroeconomic "news") $\hat{\upsilon}^G_{t+1}, \hat{\upsilon}^F_{N+1,t+1}, \hat{\upsilon}^F_{j,t+1}$ with a constant, the R^2 declines into 30% (36% or 13%). This finding is in line with several other studies. For instance, Evans (2010) finds that 23% of the variance in excess currency returns over one-month horizon can be explained by macroeconomic "news" of GDP, CPI, and M1 using weekly USD/EUR exchange rate. Evans and Lyons (2008) show that the arrival of intraday macro "news" can account for more than 30% of the variance in the daily price of the DEM/USD. Moreover, in this model, the role of macro innovations is further amplified by the time-varying market prices of risks, and hence the exchange rate dynamics are heteroskedastic. The importance of macroeconomic "news" macroeconomic has been investigated by Engel, Mark and West (2007) and Andersen et al. (2003) as well.

C. Model Performance on Macroeconomic and Yield Variables

This no-arbitrage macro-finance model is able to model exchange rate dynamics and macroeconomic and financial variables. Table 6 and Table 7 show the observed and model-implied summary statistics for macroeconomic and yield data. It is different with respect to the results for exchange rate dynamics. The model-implied macroeconomic and yield variables capture the statistics of observed ones very well, such as the mean, the standard error, the skewness, the kurtosis and the autocorrelation.

— Table 6 and 7 around here —

Moreover, Figure 6 and 7 show that the model implied macroeconomic and financial variables move tightly with observed ones. The good model fit of these two types of variables can be also deduced from the estimates of the standard deviation of measurement errors in Table 3. They are very small, with values between 0.2 and 10.8 basis points.

— Figure 6 and 7 around here —

D. Foreign Exchange Risk Premium and Forward Premium Anomaly

One of the most notable puzzles in foreign exchange markets is the forward premium anomaly. It implies that high interest rate currencies tend to appreciate. Fama (1984) attributes this departure from uncovered interest parity (UIP) to a time-varying risk premium. This model also suggests that the expected exchange rate change is equal to the sum of the interest rate differentials and the time-varying foreign exchange risk premium, which is constructed using the market prices of risks.

— Table 2 around here —

Table 2 provides the estimates of market prices of risks, where more than half of the parameters are statistically significant. Most of the estimates in $\lambda_{US,0}$ and $\lambda_{GM/UK/JP,0}$ ($\lambda_{US,1}$ and $\lambda_{GM/UK/JP,1}$) not only have the same sign, but also have very close values with respect to each other. This implies that the SDFs of the three foreign currencies should be highly correlated with the US SDF. Indeed, the correlations of the model-implied SDFs reach as high as 99%. Brandt et al. (2006) show that the volatility of the exchange rate and of the SDFs from asset markets imply that the SDFs must be highly correlated across countries. The parameters in $\lambda_{i,0}^G$ and in $\lambda_{i,0}^F$ are negative, which is consistent with previous findings (Backus et al., 1998). In addition, the parameters in $\lambda_{i,1}^G$ are much larger than in $\lambda_{i,1}^F$. The two parts of the foreign exchange risk premia (driven by global and by country-idiosyncratic factors) can been seen in Figure 4, and Figure 3 shows the sum of these two parts.

— Figure 4 around here —

Figure 4 shows that for each of these three dollar foreign exchange risk premia, the component driven by the global factors is the dominant one, since the magnitude is 100 times larger with respect to the component driven by the country-idiosyncratic factors. The component driven by the global factors have similar patterns among the three dollar exchange rates. They have three positive peaks around three monetary/financial crises, i.e., the European monetary mechanism crisis in 1992, the Asian financial crisis in 1997 and the recent financial crisis that started in 2008. Along the sample period, the European monetary mechanism crisis generates the biggest effect on the German mark and on the British pound against the US dollar, while the Asian financial crisis creates the highest peak of risk premia for the Japanese yen against the US dollar. On the contrary, the parts of foreign exchange risk premia induced by country-idiosyncratic factor have very idiosyncratic dynamic patterns among these three dollar exchange rates.

Fama (1984) argues that the implied risk premium should be negatively correlated with and have larger variance than the interest rate differentials. They are usually termed as the Fama conditions. For each of the three dollar exchange rates, this model implied risk premium (rp_t) does negatively correlate with the interest rate differentials ($r^{(h)} - r^{(f)}$), with correlations of -9%, -58%, and -22%, and has a larger variance (0.82 vs. 0.04, 0.62 vs. 0.02, and 0.84 vs. 0.03). These results are presented in Panel B, Table 5.

— Table 5 around here —

Moreover, the estimated foreign exchange risk premia are counter-cyclical to the US economy. The last panel in Table 5 it shows that foreign exchange risk premia are negatively correlated with respect to output growth differentials between the US and foreign countries. This negative correlation implies that investors in the market anticipate that the foreign currency will appreciate with respect to the domestic currency, when the foreign output growth is higher than the domestic one. When one country is in a better economic situation than the other, the market becomes more confident in that country's currency and thus investors would like to hold it, leading to the appreciation of that currency.

Similar to output growth differentials, the inflation rate differentials ($\pi^{(h)} - \pi^{(f)}$) are negatively correlated with respect to the foreign exchange risk premia, as the last panel in Table 5 shows. This means that when foreign inflation is relatively high, the foreign currency tends to appreciate with respect to the domestic currency. According to the Taylor rule, if the current inflation of the foreign country is high, people may expect the central bank to increase its interest rate in the future. This results in a decreased interest rate differential and an increased risk premium. This is consistent with the finding of Engel and West (2006) that high German inflation is associated with a strong mark with respect to the dollar. However, traditional monetary models (Frankel, 1979; Engel and Frankel, 1984) tend to predict the opposite, stating that high inflation is associated with a weak currency.

E. What Drives Exchange Rate Dynamics and foreign exchange risk premia, Global or Country-Idiosyncratic Factors?

In order to know which factors are important in driving exchange rate dynamics and foreign exchange risk premia, I implement the variance decomposition for this nonlinear exchange rate dynamic model. According to Harris and Yu (2010), given that $\Delta s_{i,t}$, $rp_{i,t}$, and $\Delta s_{i,t}^{unexp.}$ are nonlinear functions of the state vectors G_t, $F_{i,t}$, ..., $F_{N+1,t}$, the variance decompositions can be computed by using a Monte Carlo simulation conditional on filtered state factors in the sample period. First, I simulate the model by drawing random shocks υ_{t+h}^G, $\upsilon_{i,t+h}^F$, ..., $\upsilon_{N+1,t+h}^F$, (for $h = 1, 2, ..., 60$) from $N(0, I)$. Then the evolution of the state vectors can be computed by using state dynamic equations (23) and (24), and the corresponding values of Δs_i, rp_i and $\Delta s_i^{unexp.}$

can be obtained by equations (9), (11) and (12). Finally, I numerically compute the variances of forecast errors following Harris and Yu (2010) and repeat this process 1000 times to get the nonlinear variance decompositions.

The result of the variance decomposition for each type of foreign exchange rate is reported in Tables 10, 11 and 12, respectively.

— Table 10, 11 and 12 around here —

These three tables imply that global factors are much more important comparing to country-idiosyncratic factors in driving dynamics of exchange rates, foreign exchange risk premia as well as the unexpected exchange rate changes.

For exchange rate changes Δs of all three foreign exchange rates (Panel A in Tables 10, 11 and 12), the global factors explain around 60% to 70% of the short-run (1-month) forecast error variances. Their importance rises as the forecast horizon increases, and in the long-run (60-month) they explain more than 90% of forecast error variances. Among the global factors, the output growth and the interest rate are the two most important ones. In the short-run, the global interest rate is the dominant factor for the USD/GBP and the USD/JPY, and is almost equally as important as the global output growth for the USD/DEM (EUR). However, in the long-run, global output growth is the most important factor for all three dollar exchange rate dynamics. Even though country-idiosyncratic factors are less important, they do play a certain role in the short-run, accounting for approximately 30% to 40% of the forecast error variances. For the USD/DEM (EUR), the interest rate factors of the US and Germany are the two most important country-idiosyncratic factors, and the US interest rate factor explains around twice the forecast variance than the German one. For the USD/GBP (USD/JPY), the US interest rate factor and the UK (Japanese) output growth factor are the two most important ones.

Foreign exchange risk premia are almost exclusively explained by the three global factors, as the results in Panel B of Tables 10, 11 and 12 show. This is consistent with the information provided by Figure 4, which shows that the magnitude of the component driven by global factors is about 100 times larger as the one driven by country-idiosyncratic factors. For each of the three types of foreign exchange risk premia, more than two thirds of the short-run forecast error variance is driven by the global interest rate factor. However, the role of the global interest rate factor decreases, while the role of global output and inflation factors increases, as the forecast horizon increases. Almost half of the forecast variance is explained by the global output factor and one fourth by the global inflation factor in the long-run. This finding is consistent with other studies as well. For instance, Bauer and Diez (2011) find that global output growth and inflation account for about 40% of the variation in USD/EUR risk premium in 1-year forecasting horizon, while the same value is around 50% in this model.

The variance decompositions for unexpected exchange rate changes $\Delta s^{unexp.}$ have similar patterns as the ones for exchange rate changes. This is consistent with the above finding that the macroeconomic shocks play a very important role in driving exchange rate dynamics. Even though foreign exchange risk premia are exclusively driven by global factors, the role of the country-idiosyncratic factors on exchange changes cannot be ignored at all.

F. Global and Country-Idiosyncratic Macroeconomic Factors

The above section shows that global and country-idiosyncratic macroeconomic factors play very different roles in driving exchange rate dynamics and foreign exchange risk premia. Hence it is worth to investigate them.

— Table 1 around here —

Table 1 reports the estimates of parameters related to the global and country-idiosyncratic factors. In the upper panel, the factor loadings for underlying macroeconomic fundamentals on global factors are reported. Most coefficients of the global loadings are significantly different from zero. The middle panel reports the global factor dynamics. From the coefficient matrix Φ^G, I can see that each global macroeconomic factor is highly persistent, with the diagonal values being close to one. The t-statistic in these two panels imply that global macroeconomic factors do exist.

The bottom panel in Table 1 presents the dynamics of the country-idiosyncratic factors. Diagonal values of matrix Φ^{F_i} $(i = 1, 2, 3, 4)$ are significantly different from zero. Hence the role of the country-idiosyncratic factors are not negligible in determining the underlying macroeconomic fundamentals X_i. In addition, these values are close to one, especially the values of the coefficients for output growth. This implies that country-idiosyncratic factors are very persistent.

— Figure 8 around here —

Figure 8 plots the three global macroeconomic factors, i.e. output growth, inflation and interest rate. There are three big slumps for global output growth around the years of 1992, 1997 and 2008, when there were the European monetary mechanism crisis, the Asian financial crisis and the recent US financial crisis. The global inflation factor is positive along the sample period except for the time around 2009. The global short-term interest rate factor has the highest peak around 1991, which is exactly the same time of the peak of short-term interest rates (Figure 1).

— Figure 9 around here —

Figure 9 plots country-idiosyncratic output growth, inflation and interest rate factors in the top, middle and bottom panel, respectively. In each panel, there are four country-idiosyncratic factors for Germany, the UK, Japan and the US, respectively. Compared to the macroeconomic fundamentals in Figure 1, country-idiosyncratic factors show much much less cross-country comovement.

VI. Conclusion

This paper simultaneously investigates the dynamics of multiple bilateral nominal exchange rates using a no-arbitrage multi-country model. I employ macroeconomic fundamental information to model exchange rate dynamics from a no-arbitrage macro-finance perspective. Macroeconomic fundamentals are assumed to be determined by both global (common) factors and country-idiosyncratic factors.

The empirical study focuses on an open economy including four countries, i.e. Germany, the UK, Japan and the US, where the US is taken as the home country. The empirical evidence shows that this no-arbitrage multi-country model is able to well characterize the joint dynamics of all three major dollar exchange rates. The model-implied monthly exchange rate changes can explain 57%, 66% and 33% of the variations of the observed changes of USD/DEM (EUR), USD/GBP and USD/JPY, respectively. The model-implied foreign exchange risk premia satisfy the Fama conditions (1984) and they are counter cyclical with respect to the US economy. The macroeconomic innovations, or “news”, are important in determining exchange rate dynamics. Moreover, the global and country-idiosyncratic macroeconomic factors do exist and play very different roles in driving exchange rate dynamics and foreign exchange risk premia. Global factors drive foreign exchange risk premia almost exclusively and account for more than half of the variation of exchange rate dynamics, in both the short and long run. In the short run the global interest rate is the dominant factor, while in the long run global output becomes dominant in driving exchange rate dynamics and foreign exchange risk premia. Even though country-idiosyncratic macroeconomic factors are less important compared to global ones, they do play a role in short-run exchange rate dynamics, notably interest rate factors from the US and German, and output growth factors from the UK and Japanese.

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Table 1: **Estimates of the Global and Country-Idiosyncratic Factor Parameters**

Factor Loadings ($X_{i,t} = \alpha_i + \beta_i G_t + F_{i,t}$)						
	$\alpha_i(\times 10^3)$			β_i		
	α_i^g	α_i^π	α_i^r	β_i^g	β_i^π	β_i^r
GM	4.59	-0.14	-0.08	0.62	0.16	0.72
	(2.26)	(2.83)	(2.20)	(1.86)	(1.98)	(4.46)
UK	-0.10	0.18	0.14	0.31	0.26	0.85
	(1.81)	(2.68)	(2.74)	(2.61)	(3.74)	(4.01)
JP	0.15	-0.59	0.23	0.94	0.22	0.57
	(3.09)	(2.99)	(3.28)	(2.96)	(2.59)	(5.27)
US	0.46	0.26	0.01	0.18	0.25	0.76
	(1.94)	(2.28)	(3.02)	(3.67)	(2.70)	(4.03)
Global Factor Dynamics ($G_t = \Phi^G G_{t-1} + \Sigma^G \upsilon_t^G$)						
	Φ^G			$\Sigma^G(\times 10^3)$		
	g^G	π^G	r^G	g^G	π^G	r^G
g^G	0.98	-0.01	-0.02	1	0	0
	(48.45)	(3.64)	(2.82)	–	–	–
π^G	0.15	0.90	0.07	0	1	0
	(2.40)	(27.40)	(1.64)	–	–	–
r^G	-0.16	0.08	0.89	0	0	1
	(2.53)	(4.90)	(16.16)	–	–	–
Country-Idiosyncratic Factor Dynamics ($F_{i,t} = \Phi^{F_i} F_{i,t-1} + \Sigma^{F_i} \upsilon_{i,t}^F$)						
	Φ^{F_i} (*diagonal*)			Σ^{F_i} ($\times 10^3$, *diagonal*)		
	f_i^g	f_i^π	f_i^r	f_i^g	f_i^π	f_i^r
GM	0.98	0.99	0.94	1.07	0.26	0.25
	(16.70)	(73.16)	(160.39)	(3.74)	(7.47)	(6.43)
UK	0.99	0.99	0.87	0.73	0.33	0.42
	(54.54)	(66.79)	(62.35)	(5.37)	(9.31)	(6.39)
JP	0.98	0.96	0.99	1.34	0.28	0.15
	(52.73)	(32.75)	(145.85)	(1.85)	(5.99)	(5.14)
US	0.99	0.98	0.93	0.65	0.26	0.33
	(93.88)	(38.83)	(133.83)	(3.80)	(2.66)	(5.90)

Note: This table reports the estimates of the global and country-idiosyncratic factor parameters. In parentheses, the absolute value of t-ratio of each estimate is reported. The sample period is from 1985m01 to 2009m05 (293 observations).

Table 2: **Estimates of Market Price of Risk Parameters**

	$\lambda_{i,0}^{G}$ $(\times 10^2)$			$\lambda_{i,0}^{F}$ $(\times 10^2)$		
	g^G	π^G	r^G	f_i^g	f_i^π	f_i^r
GM	-4.45	-7.81	-6.09	-0.39	-0.14	-0.82
	(2.76)	(4.45)	(3.13)	(1.92)	(1.22)	(1.29)
UK	-4.37	-9.51	-2.95	-1.10	-0.14	0.48
	(3.35)	(3.90)	(2.85)	(1.84)	(3.00)	(3.12)
JP	-3.84	-9.55	-2.04	-1.84	-0.41	-0.25
	(5.45)	(2.77)	(2.03)	(5.09)	(1.42)	(3.30)
US	-1.61	-7.75	-4.56	-0.60	-0.40	-1.33
	(2.28)	(2.42)	(3.83)	(4.20)	(2.27)	(4.14)
	$\lambda_{i,1}^{G}$ $(diagonal)$			$\lambda_{i,1}^{F}$ $(diagonal)$		
	g^G	π^G	r^G	f_i^g	f_i^π	f_i^r
GM	39.37	-25.65	-34.44	1.46	1.98	-6.62
	(7.30)	(5.96)	(8.68)	(1.74)	(1.83)	(2.69)
UK	39.89	-23.63	-37.71	3.63	-1.12	-6.90
	(7.90)	(5.17)	(9.49)	(2.38)	(1.36)	(1.70)
JP	38.53	-22.79	-39.11	2.69	-1.79	-4.44
	(7.27)	(3.98)	(7.68)	(2.68)	(1.32)	(3.41)
US	42.18	-24.10	-40.96	1.44	-2.86	-2.77
	(7.59)	(6.16)	(9.35)	(1.89)	(2.02)	(1.96)

Note: This table reports the estimates of market price of risk parameters. In parentheses, the absolute value of t-ratio of each estimate is reported. The sample period is from 1985m01 to 2009m05 (293 observations).

Table 3: **Estimates of Standard Deviation of Measurement Error Parameters** ($\times 10^4$)

	g_i	π_i	r_i	$y_i^{(24)}$	$y_i^{(60)}$	Δs_j
GM	10.81	0.34	3.75	0.26	1.77	212.27
	(4.65)	(2.87)	(7.84)	(3.49)	(3.10)	(2.48)
UK	6.56	0.67	6.62	0.09	0.88	202.52
	(3.70)	(2.59)	(4.60)	(1.91)	(1.95)	(3.64)
JP	9.70	1.31	1.56	0.79	0.99	271.85
	(2.12)	(3.73)	(2.82)	(3.60)	(4.52)	(7.58)
US	0.55	0.28	5.64	0.66	2.80	
	(2.89)	(3.57)	(6.73)	(2.18)	(2.94)	

Note: This Table reports the estimates of standard deviation of measurement error parameters. In parentheses, the absolute value of t-ratio of each estimate is reported. The sample period is from 1985m01 to 2009m05 (293 observations).

Table 4: **Model Performance: Exchange Rate Dynamics**

	Mean(%)	*Std. Dev.*(%)	*Skewness*	*Kurtosis*	*Autocorr.*
			1. USD/GEM (EUR)		
Data	0.28	3.22	-0.24	3.92	0.07
Model	0.02	2.59	-0.27	4.93	0.09
			2. USD/GBP		
Data	0.12	3.05	-0.28	5.95	0.11
Model	-0.09	2.54	-0.41	5.89	0.14
			3. USD/JPY		
Data	0.33	3.24	0.30	4.43	0.08
Model	0.17	2.49	-0.97	8.57	0.13

Note: This table reports model performance for exchange rate dynamics. The sample period is from 1985m01 to 2009m05 (293 observations).

Table 5: **Model-implied Exchange Rate Dynamics and foreign exchange risk premia**

Panel A. Δs and $\hat{\Delta s}$			
	USD/DEM	*USD/GBP*	*USD/JPY*
Explained Variation (R^2,%)	57	66	33
Corr(Δs,$\hat{\Delta s}$)(%)	75	81	58
Panel B. Fama Conditions			
	USD/DEM	*USD/GBP*	*USD/JPY*
Corr(rp,$r^{(h)} - r^{(f)}$)(%)	-9	-58	-22
Var(rp) ($\times 10^4$)	0.82	0.62	0.84
Var($r^{(h)} - r^{(f)}$) ($\times 10^4$)	0.04	0.02	0.03
Panel C. foreign exchange risk premia and Macro Differentials			
	USD/DEM	*USD/GBP*	*USD/JPY*
Corr(rp,$g^{(h)} - g^{(f)}$)(%)	-11	-32	-4
Corr(rp,$\pi^{(h)} - \pi^{(f)}$)(%)	-13	-59	-27

Note: This table reports model fitting for exchange rate dynamics in Panel A, the Fama Conditions in Panel B and the correlations between foreign exchange risk premia and macroeconomic differentials in Panel C. The sample period is from 1985m01 to 2009m05 (293 observations).

Table 6: **Model Performance: Macroeconomic Variables**

		Mean(%)	*Std. Dev.*(%)	*Skewness*	*Kurtosis*	*Autocorr.*
1. Germany						
output growth	Data	0.14	0.42	-2.07	10.51	0.86
	Model	0.15	0.40	-2.24	10.65	0.92
inflation	Data	0.16	0.11	0.85	4.34	0.96
	Model	0.16	0.11	0.85	4.34	0.96
interest rate	Data	0.38	0.18	0.84	2.93	0.98
	Model	0.38	0.18	1.02	3.21	0.99
2. UK						
output growth	Data	0.07	0.27	-1.02	6.85	0.88
	Model	0.07	0.26	-1.09	6.99	0.93
inflation	Data	0.31	0.17	1.24	4.79	0.97
	Model	0.31	0.17	1.24	4.79	0.97
interest rate	Data	0.61	0.28	0.76	2.73	0.98
	Model	0.59	0.25	0.57	2.65	0.97
3. Japan						
output growth	Data	0.07	0.54	-2.32	11.82	0.92
	Model	0.07	0.53	-2.37	12.01	0.93
inflation	Data	0.06	0.10	0.73	2.71	0.95
	Model	0.06	0.10	0.72	2.67	0.96
interest rate	Data	0.18	0.21	0.93	2.49	0.99
	Model	0.17	0.20	0.90	2.41	0.99
4. US						
output growth	Data	0.19	0.29	-1.41	6.76	0.94
	Model	0.19	0.29	-1.41	6.76	0.94
inflation	Data	0.25	0.10	0.01	3.63	0.93
	Model	0.25	0.10	0.01	3.63	0.93
interest rate	Data	0.37	0.17	-0.21	2.41	0.98
	Model	0.41	0.19	-0.15	2.47	0.97

Note: This table reports the statistic summary of observed and model-implied macroeconomic variables. The sample period is from 1985m01 to 2009m05 (293 observations).

Table 7: **Model Performance: Yield Data**

Maturities		*Mean(%)*	*Std. Dev.(%)*	*Skewness*	*Kurtosis*	*Autocorr.*
1. Germany						
24-m	Data	0.40	0.16	0.79	2.88	0.98
	Model	0.40	0.16	0.78	2.88	0.98
60-m	Data	0.44	0.14	0.50	2.48	0.98
	Model	0.44	0.15	0.49	2.31	0.98
2. UK						
24-m	Data	0.57	0.22	0.44	2.32	0.98
	Model	0.57	0.22	0.44	2.32	0.98
60-m	Data	0.58	0.20	0.33	1.86	0.98
	Model	0.59	0.20	0.31	1.86	0.98
3. Japan						
24-m	Data	0.19	0.19	0.84	2.34	0.99
	Model	0.19	0.19	0.81	2.28	0.99
60-m	Data	0.23	0.18	0.61	1.98	0.99
	Model	0.23	0.18	0.66	2.08	0.99
4. US						
24-m	Data	0.44	0.18	-0.12	2.41	0.97
	Model	0.44	0.18	-0.11	2.42	0.97
60-m	Data	0.49	0.16	0.18	2.49	0.97
	Model	0.49	0.17	-0.03	2.07	0.98

Note: This table reports the statistic summary of observed and model-implied yield variables. The sample period is from 1985m01 to 2009m05 (293 observations).

Table 8: **Data Correlations: Macro and Exchange Rate Data**

	01	*02*	*03*	*04*	*05*	*06*	*07*	*08*	*09*	*10*	*11*	*12*	*13*	*14*	*15*
01	1.00														
02	0.15	1.00													
03	0.53	0.32	1.00												
04	0.14	0.53	0.30	1.00											
05	-0.25	-0.19	-0.22	-0.16	1.00										
06	0.49	0.01	0.37	-0.17	0.20	1.00									
07	0.20	-0.07	0.11	-0.08	0.55	0.70	1.00								
08	0.34	0.04	0.49	-0.12	0.35	0.78	0.58	1.00							
09	-0.03	-0.15	-0.12	-0.24	0.76	0.56	0.73	0.56	1.00						
10	0.36	0.12	0.24	-0.18	0.19	0.88	0.72	0.68	0.65	1.00					
11	0.08	-0.01	0.09	-0.27	0.44	0.71	0.77	0.61	0.82	0.84	1.00				
12	0.48	0.35	0.36	0.14	-0.04	0.67	0.47	0.60	0.42	0.80	0.54	1.00			
13	0.03	0.03	-0.02	-0.05	-0.02	0.07	0.03	-0.04	0.03	0.10	0.10	-0.00	1.00		
14	0.02	0.01	0.03	0.02	-0.06	0.01	-0.01	-0.03	-0.06	0.02	0.01	-0.02	0.72	1.00	
15	-0.05	0.10	-0.01	0.00	-0.03	0.01	0.03	-0.02	0.03	0.05	0.12	-0.01	0.54	0.44	1.00

Note: This table reports the correlations of original monthly macroeconomic variables and exchange rate changes. There are four time series of output growth rates (index of *1-4*), inflation rates (index of *5-8*) and short-term interest rates (index of *9-12*), which are for Germany, the UK, Japan, and the US, repectively. Exchange rate changes data (index of *13-15*) are respectively the Germany mark/euro, the British proud and the Japanese yen, against the US dollar. The exchange rate changes are the monthly changes of log nominal exchange rate (the end of period market rate). The output growth rates and inflation rates are the annualized changes of IP and CPI, respectively. The sample period is from 1985m01 to 2009m05 (293 observations).

Table 9: **Principal Component Analysis for Macroeconomic Fundamentals**

I. Output Growth				
	PC1	*PC2*	*PC3*	*PC4*
Variance prop.	76	12	8	4
Cumulative prop.	76	88	96	100
II. Inflation Rates				
	PC1	*PC2*	*PC3*	*PC4*
Variance prop.	71	18	7	3
Cumulative prop.	71	89	97	100
III. Short-Term Interest Rates				
	PC1	*PC2*	*PC3*	*PC4*
Variance prop.	84	10	4	2
Cumulative prop.	84	94	98	100

Note: This table reports the preliminary analysis of principle component analysis for macroeconomic fundamentals. For each group of output growth rates, inflation rates, and short-term interest rates, I report the variance proportions and cumulative variance proportions in percentage associated with the four principal components, which are positioned with a descending order according to associated eigenvalues. The sample period is from 1985m01 to 2009m05 (293 observations).

Table 10: **Variance Decompositions: USD/DEM (EUR)**

	Global: G			*US*:F_4			*Germany*: F_1		
N	g^G	π^G	r^G	f_4^g	f_4^π	f_4^r	f_1^g	f_1^π	f_1^r
				Panel A. Exchange rate changes, Δs					
1	31.47	4.41	27.81	2.29	4.35	15.80	3.80	0.39	9.67
3	31.82	4.36	29.57	2.28	4.15	14.95	3.77	0.41	8.68
12	39.33	4.35	28.64	2.32	3.20	11.58	3.74	0.41	6.42
24	48.44	5.46	29.87	1.83	1.55	6.60	2.70	0.30	3.25
60	51.69	10.33	30.83	0.98	0.62	2.80	1.26	0.14	1.36
				Panel B. foreign exchange risk premium, rp					
1	22.59	4.41	72.98	0.00	0.00	0.00	0.00	0.00	0.01
3	25.69	3.26	71.04	0.00	0.00	0.00	0.00	0.00	0.01
12	41.89	11.47	46.63	0.00	0.00	0.00	0.00	0.00	0.00
24	49.43	18.07	32.50	0.00	0.00	0.00	0.00	0.00	0.00
60	49.59	21.28	29.13	0.00	0.00	0.00	0.00	0.00	0.00
				Panel C. Unexpected changes, $\Delta s^{unexp.}$					
1	31.46	4.42	27.64	2.30	4.37	15.88	3.82	0.39	9.71
3	31.83	4.38	29.22	2.30	4.19	15.10	3.80	0.42	8.76
12	38.85	4.09	28.20	2.42	3.35	12.05	3.91	0.44	6.69
24	48.20	2.26	29.76	2.24	1.90	8.01	3.28	0.37	3.98
60	52.74	1.59	33.54	1.66	1.05	4.76	2.13	0.24	2.31

Note: This table reports variance decompositions for the model-implied exchange rate changes, the risk premium and the unexpected changes of the USD/DEM (EUR). The forecast horizons (N) are in months. The sample period is from 1985m01 to 2009m05 (293 observations).

Table 11: **Variance Decompositions: USD/GBP**

	Global: G			*US*:F_4			*UK*: F_2		
N	g^G	π^G	r^G	f_4^g	f_4^π	f_4^r	f_2^g	f_2^π	f_2^r
			Panel A. Exchange rate changes, Δs						
1	20.22	11.22	47.11	1.29	2.55	8.88	5.69	0.50	2.53
3	20.69	11.28	46.70	1.35	2.58	8.81	5.85	0.50	2.24
12	24.61	11.12	42.36	1.54	2.28	8.09	7.46	0.50	2.04
24	36.37	11.13	32.80	1.65	1.45	5.92	8.37	0.35	1.96
60	45.64	13.70	27.81	1.22	0.76	3.40	6.17	0.19	1.12
			Panel B. foreign exchange risk premium, rp						
1	20.24	1.95	77.78	0.00	0.00	0.00	0.02	0.00	0.01
3	23.09	1.58	75.30	0.00	0.00	0.00	0.02	0.00	0.01
12	37.81	14.90	47.25	0.00	0.00	0.00	0.03	0.00	0.00
24	47.68	20.43	31.87	0.00	0.00	0.00	0.01	0.00	0.00
60	48.90	22.74	28.35	0.00	0.00	0.00	0.00	0.00	0.00
			Panel C. Unexpected changes, $\Delta s^{unexp.}$						
1	20.23	11.23	47.06	1.29	2.56	8.89	5.71	0.50	2.52
3	20.67	11.32	46.61	1.36	2.59	8.83	5.88	0.51	2.24
12	24.03	11.10	42.49	1.58	2.33	8.26	7.64	0.51	2.06
24	34.37	9.47	33.81	1.88	1.67	6.73	9.47	0.40	2.21
60	44.41	7.57	29.00	1.80	1.14	5.08	9.08	0.28	1.64

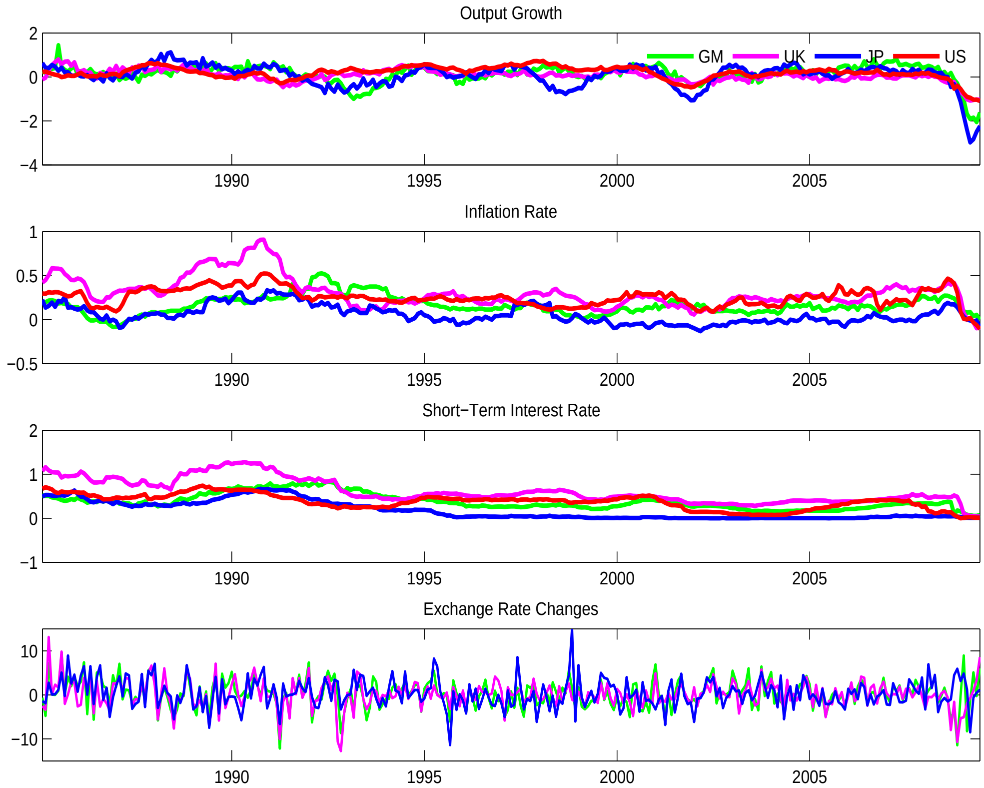
Note: This table reports variance decompositions for the model-implied exchange rate changes, the risk premium and the unexpected changes of the USD/GBP. The forecast horizons (N) are in months. The sample period is from 1985m01 to 2009m05 (293 observations).

Table 12: **Variance Decompositions: USD/JPY**

N	*Global*: G			*US*:F_4			*Japan*: F_3		
	g^G	π^G	r^G	f_4^g	f_4^π	f_4^r	f_3^g	f_3^π	f_3^r
				Panel A. Exchange rate changes, Δs					
1	18.10	6.77	51.50	1.09	2.30	8.25	9.76	0.89	1.34
3	19.51	6.57	50.31	1.13	2.28	7.95	10.12	0.88	1.25
12	27.51	6.79	42.02	1.29	1.87	6.83	11.87	0.77	1.05
24	44.26	9.08	28.66	1.20	1.02	4.29	10.44	0.46	0.59
60	54.18	13.42	22.23	0.75	0.48	2.15	6.29	0.23	0.28
				Panel B. foreign exchange risk premium, rp					
1	30.01	0.60	69.33	0.00	0.00	0.00	0.06	0.00	0.00
3	32.36	1.46	66.12	0.00	0.00	0.00	0.06	0.00	0.00
12	45.55	15.55	38.82	0.00	0.00	0.00	0.07	0.00	0.00
24	51.28	21.23	27.47	0.00	0.00	0.00	0.02	0.00	0.00
60	50.57	24.00	25.42	0.00	0.00	0.00	0.00	0.00	0.00
				Panel C. Unexpected changes, $\Delta s^{unexp.}$					
1	18.07	6.79	51.44	1.10	2.31	8.28	9.78	0.89	1.35
3	19.40	6.60	50.25	1.14	2.30	7.98	10.18	0.89	1.26
12	26.99	6.58	42.06	1.33	1.93	7.02	12.23	0.80	1.07
24	44.27	5.96	28.42	1.42	1.23	5.09	12.36	0.55	0.69
60	60.30	4.57	18.84	1.19	0.77	3.46	10.05	0.38	0.44

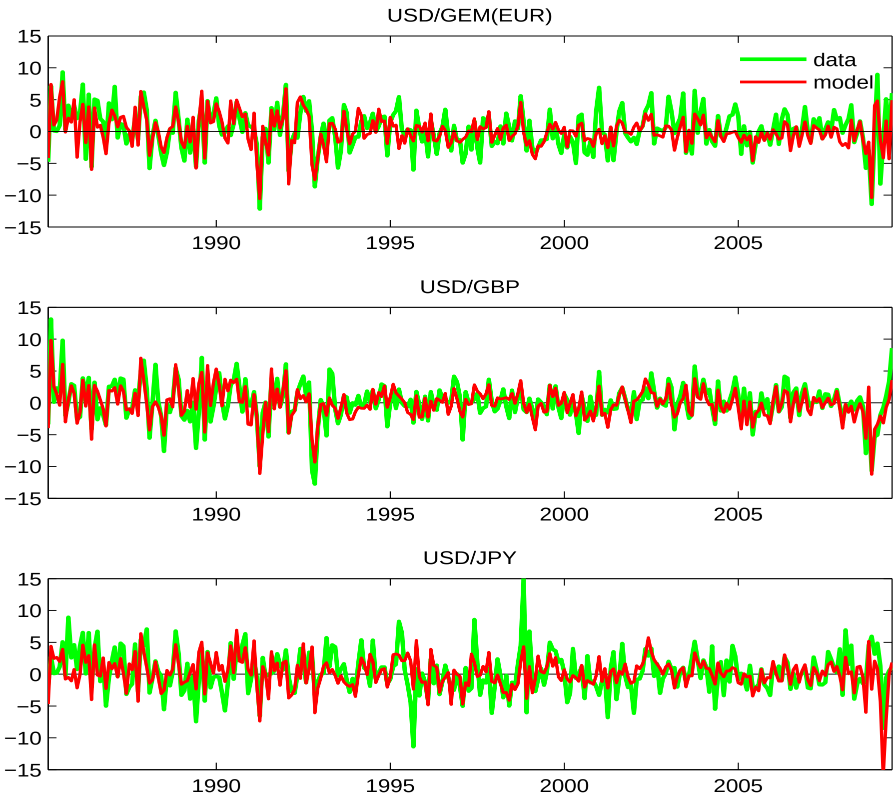
Note: This table reports variance decompositions for the model-implied exchange rate changes, the risk premium and the unexpected changes of the USD/JPY. The forecast horizons (N) are in months. The sample period is from 1985m01 to 2009m05 (293 observations).

Figure 1: Macroeconomic Data and Exchange Rates Changes



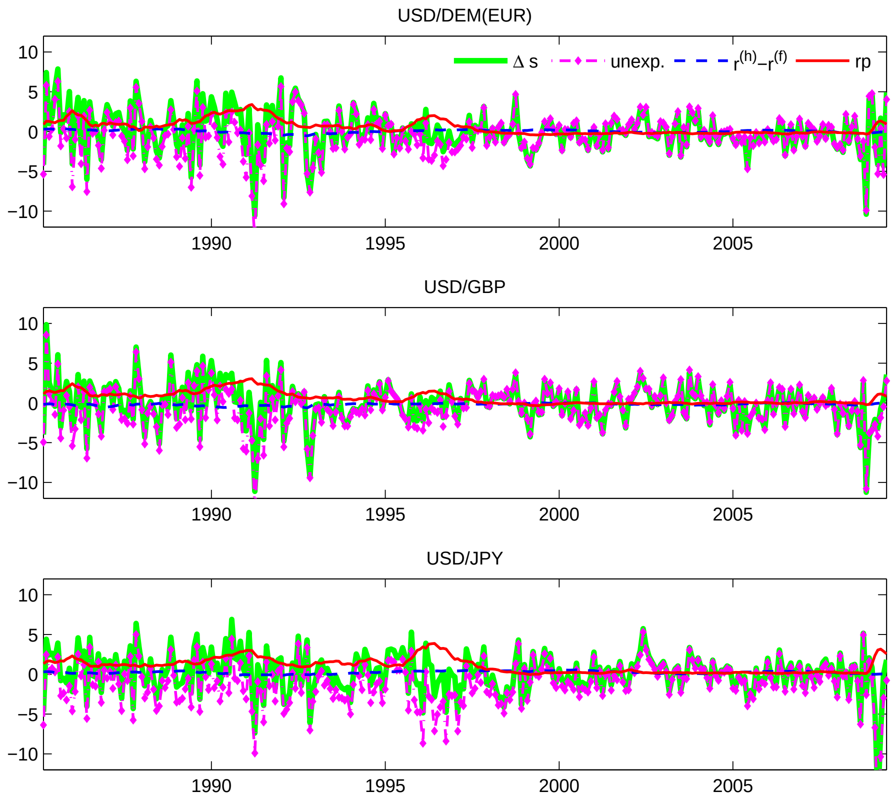
Note: This figure plots the macroeconomic fundamentals and monthly changes of exchange rates used in the estimation, for Germany, the UK, Japan, and the US. The panels are for output growth rates, inflation rates, short-term interest rates and exchange rate changes, respectively. Both macroeconomic variables and exchange rates changes are in monthly equalized quantities. The sample period is from 1985m01 to 2009m05 (293 observations).

Figure 2: Exchange Rate Dynamics



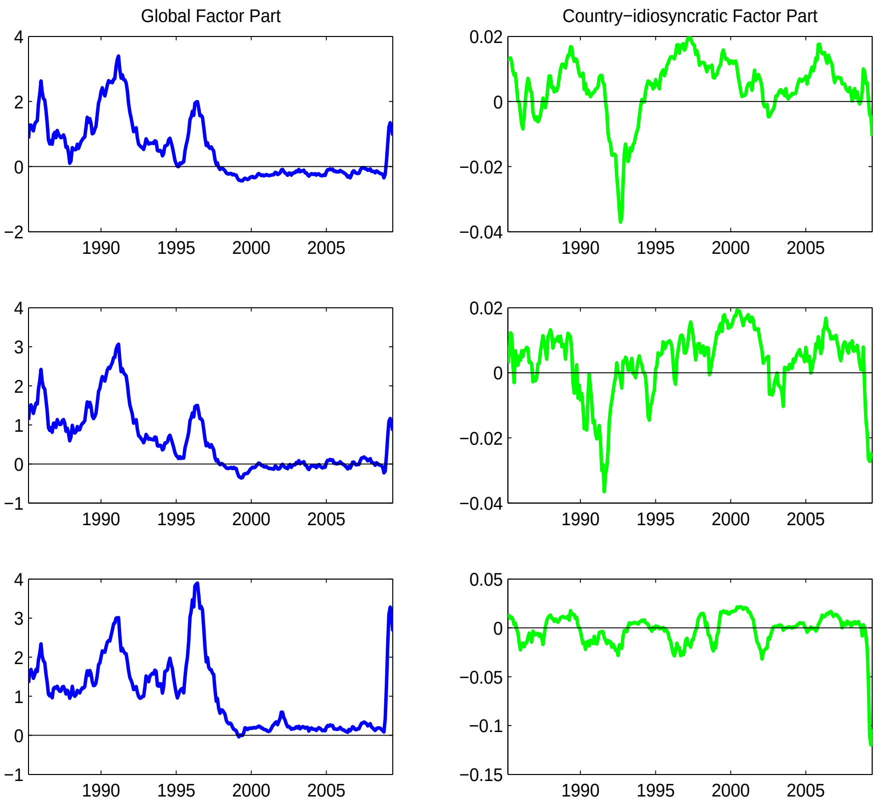
Note: This figure plots the monthly exchange rate changes in percentage of the foreign currencies, such as, Germany, the UK, and Japan, against the US dollar. The thick lines are observed data, while the thin lines are model-implied ones. The model is able to capture 57%, 66% and 33% of the observed exchange rates variances for the USD/GEM (EUR), the USD/GBP and the USD/JPY, respectively. The sample period is from 1985m01 to 2009m05 (293 observations).

Figure 3: Model-implied Exchange Rate Dynamics and Their Components



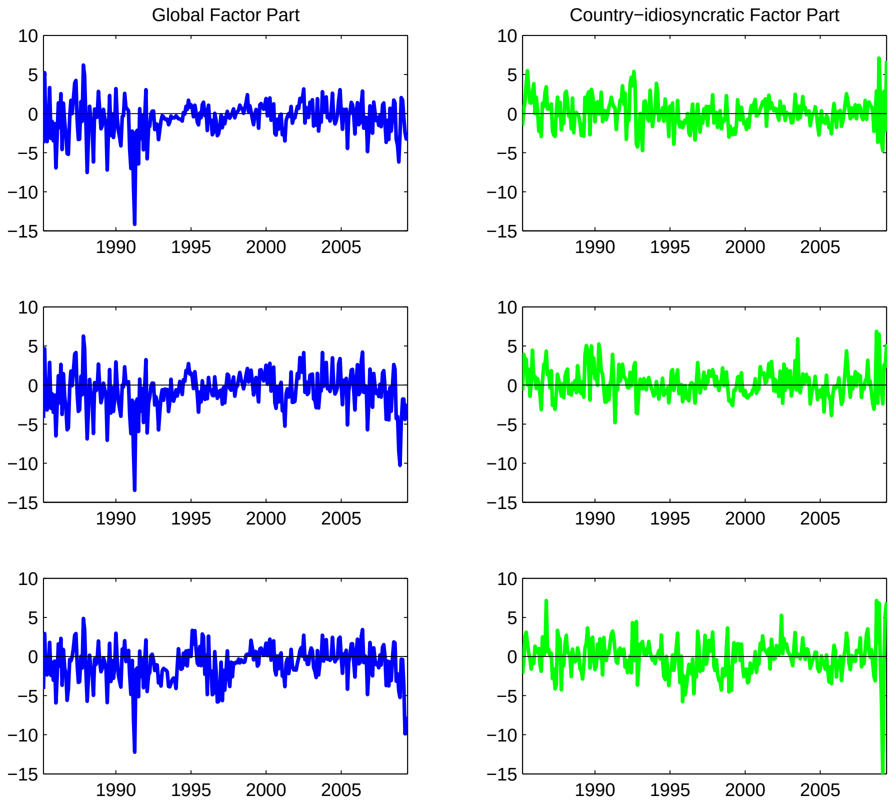
Note: This figure plots the model-implied monthly exchange rate changes, unexpected exchange rate changes, short-term interest rate differentials as well as foreign exchange risk premia, in percentage for the foreign currencies, such as, Germany, the UK, and Japan, against the US dollar. The thick light lines are model-implied exchange rate changes, the thin dark lines are model-implied foreign exchange risk premia, while the dash-dot lines are short-term interest rate differentials. The sample period is from 1985m01 to 2009m05 (293 observations).

Figure 4: Model-implied Foreign Exchange Risk Premia by Global and by Country-idiosyncratic Factors



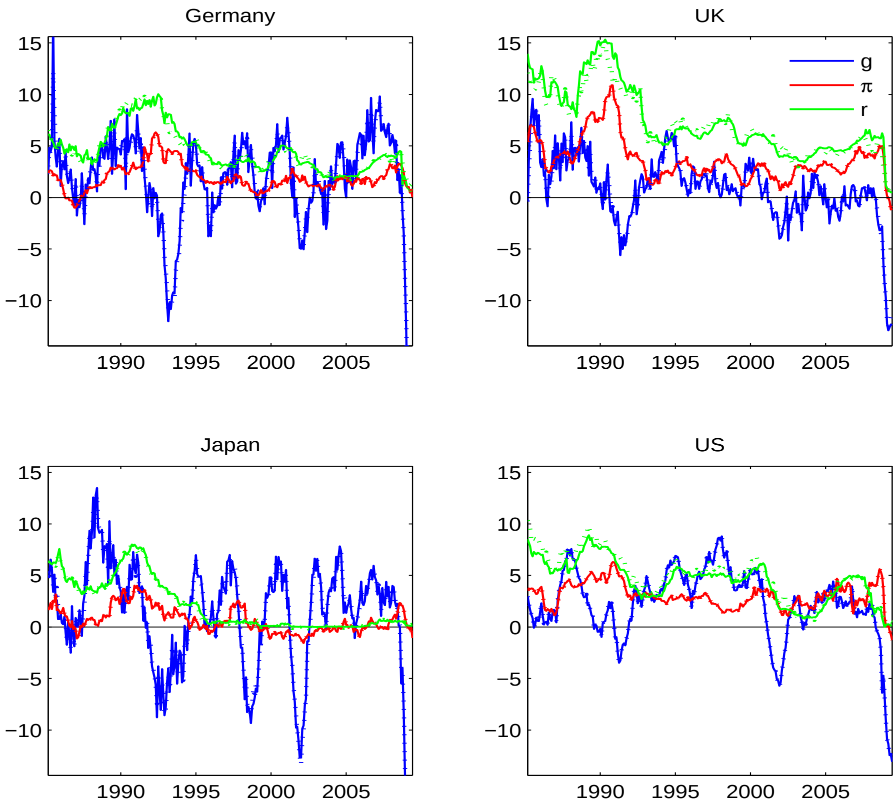
Note: This figure plots the model-implied foreign exchange risk premia driving by global factors (the left panels) and country-idiosyncratic factors (the right panels) in percentage. Pictures from the top to the bottom panels are for the USD/DEM(EUR), the USD/GBP and the USD/JPY. The sample period is from 1985m01 to 2009m05 (293 observations).

Figure 5: Model-implied Unexpected Exchange Rate Dynamics by Global and by Country-idiosyncratic Factors



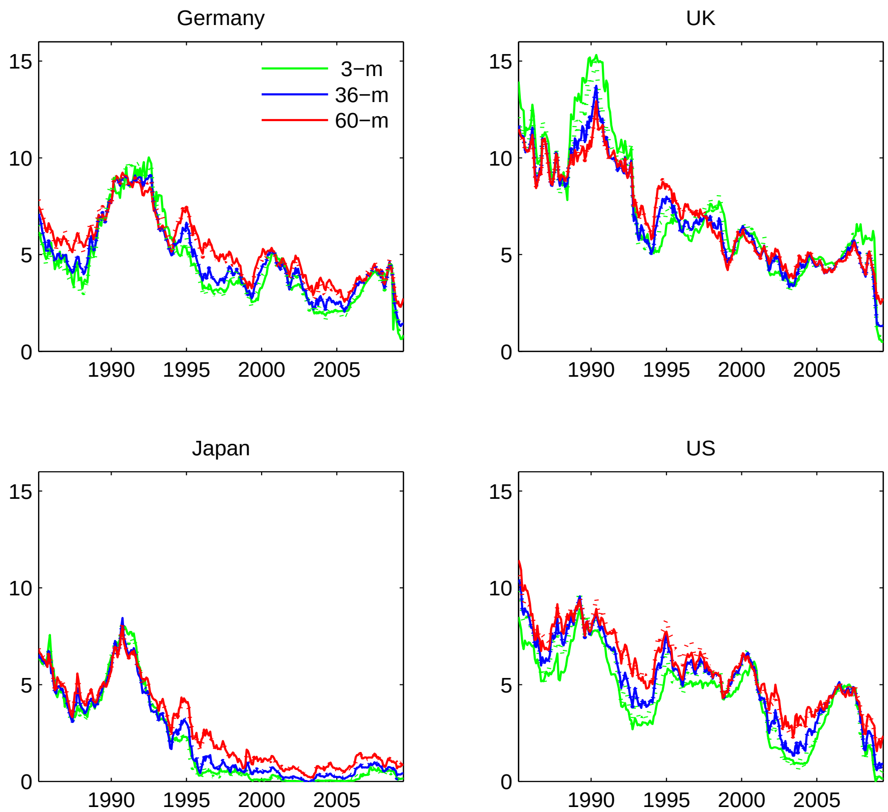
Note: This figure plots the model-implied unexpected exchange rate dynamics driving by global factors (the left panels) and country-idiosyncratic factors (the right panels) in percentage. Pictures from the top to the bottom panels are for the USD/DEM(EUR), the USD/GBP and the USD/JPY. The sample period is from 1985m01 to 2009m05 (293 observations).

Figure 6: Observed and Model-implied Macroeconomic Fundamentals



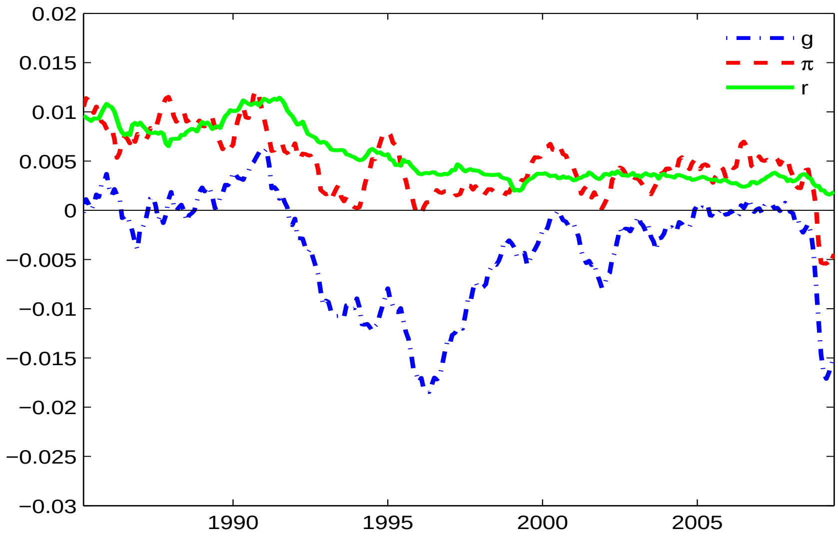
Note: This figure plots the monthly observed (solid lines) and model-implied (dotted lines) macroeconomic fundamentals, i.e. output growth, inflation rate, short-term interest rate, for Germany, the UK, Japan and the US. The sample period is from 1985m01 to 2009m05 (293 observations).

Figure 7: Observed and Model-implied Yield Data



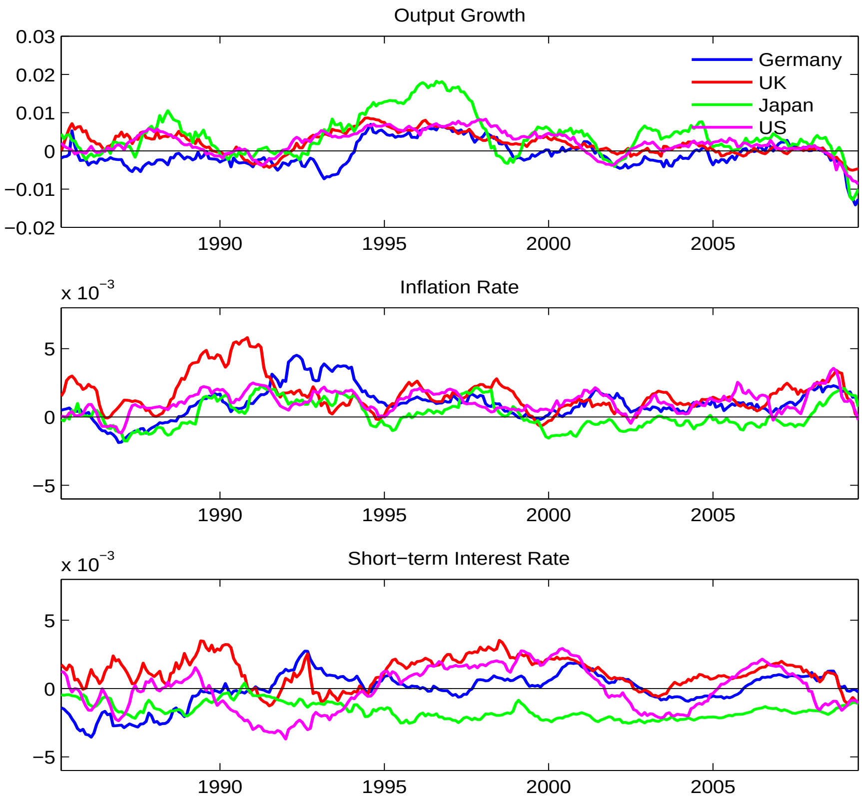
*Note:*This figure plots the monthly observed (solid lines) and model-implied (dotted lines) yield data, with maturities of 3-month, 24-month and 60-month, for Germany, the UK, Japan and the US. The sample period is from 1985m01 to 2009m05 (293 observations).

Figure 8: Global Macroeconomic Factors



Note: This figure plots the global macroeconomic factors filtered from the no-arbitrage multi-country model. The sample period is from 1985m01 to 2009m05 (293 observations).

Figure 9: Country-Idiosyncratic Macroeconomic Factors



Note: This figure plots the country-idiosyncratic macroeconomic factors filtered from the no-arbitrage multi-country model, for Germany, the UK, Japan and the US, respectively. The sample period is from 1985m01 to 2009m05 (293 observations).